1963

An analysis of the properties of two-dimensional incompressible fluid flow in the mixing chamber of a constant area ejector

Kiefer, Martin D.

Monterey, California: U.S. Naval Postgraduate School

AN ANALYSIS OF THE PROPERTIES OF TWO-DIMENSIONAL INCOMPRESSIBLE FLUID FLOW IN THE MIXING CHAMBER OF A CONSTANT AREA EJECTOR

MARTIN D. KIEFER

AN ANALYSIS OF THE PROPERTIES OF TWO-DIMENSIONAL INCOMPRESSIBLE FLUID FLOW IN THE MIXING CHAMBER OF A CONSTANT AREA EJECTOR

* * * * *

Martin D. Kiefer

AN ANALYSIS OF THE PROPERTIES OF TWO-DIMENSIONAL INCOMPRESSIBLE FLUID FLOW IN THE MIXING CHAMBER OF A CONSTANT AREA EJECTOR

by

Martin D. Kiefer

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1 9 6 3

Archive

B

fer, m.

AN ANALYSIS OF THE PROPERTIES OF TWO-DIMENSIONAL INCOMPRESSIBLE FLUID FLOW IN THE MIXING CHAMBER OF A CONSTANT AREA EJECTOR

by

Martin D. Kiefer

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING
from the
United States Naval Postgraduate School

ABSTRACT

The properties of an incompressible fluid in the mixing chamber of a constant area cylindrical ejector are analyzed in a two-dimensional form. The flow field for velocity and temperature is assumed to be made up of regions in which uniform flow of the primary and secondary fluids exist, and which diminish with axial distance from the ejector entrance. The velocity and temperature distributions in the mixed region are assumed to have cosine shaped profiles. The compatible solutions are given an axial distribution by assuming a parabolic spread rate for the secondary fluid jet boundary.

Results are generated in the form of a non-dimensional velocity and pressure, a normalized temperature and a Momentum Factor, as functions of axial distance. Area ratios studied include 100:1, 9:1 and 2.25:1. Velocity ratios studied include 50:1, 10:1, 3:1 and 1.5:1. The Fortran programs employed to generate compatible solutions, titled EJECTMIX I and EJECTMIX II, are included.

ACKNOWLEDGEMENT

The writer wishes to express his appreciation for the assistance and encouragement given him by Professor Paul F. Pucci of the United States Naval Postgraduate School in this investigation.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

Fortran Symbol	Mathematical Symbol	Definition
ALPHA	$\frac{\dot{m}}{\pi \rho V_{s_o} r_{a_o}^2}$	numerical value of mass rate of flow at the entrance to the mixing chamber
ANA	$\frac{r_a - r_{at}}{r_{bt} - r_{at}} \pi$	function angles for numerical integration of total enthalpy
ANB	$\frac{r_b - r_{at}}{r_{bt} - r_{at}} \pi$	function angles for numerical integration of total enthalpy
ANC	0	function angles for numerical integration of total enthalpy
BETA	$\frac{\text{Momentum Rate}}{\pi \rho V_{s_o}^2 r_{a_o}^2}$	numerical value of the momentum rate at the entrance to the mixing chamber
CHI	$\frac{r_b - r_a}{\pi}$	a constant of integration for continuity and momentum equations
CHIA	$\frac{r_{at} - r_a}{\pi}$	constants of integration for energy equation
CHIB	$\frac{r_{bt} - r_{at}}{\pi}$	constants of integration for energy equation
CONST	$\frac{(T_p - \bar{T})\bar{V}}{(V_p - \bar{V})\bar{T}}$	proportionality constant between axial fluid temperature and axial fluid velocity after all spreading radii have reached their respective limits
DA	$\frac{(r_b - r_a)\pi}{100 r_s}$	incremental changes in the angles during numerical integration of total enthalpy rate
DB	$\frac{(r_b - r_a)}{100(r_b - r_a)}$	incremental changes in the angles during numerical integration of total enthalpy rate
DELTA		numerical value of Bernoulli's constant for secondary fluid

DIST(I).... $\frac{X}{r_{a_o}}$ .... non-dimensional ratio of axial distance from the entrance to the mixing chamber to that of the peripheral diameter of the central jet

DP(I)...... $\frac{\Delta p}{\frac{1}{2}\rho V_{s_o}^2}$ .... the change in the non-dimensional value of P(I) between a given cross-section and the entrance to the mixing chamber

DT......... $\frac{T_p - T_s}{2}$ .... one-half of the difference between the central and peripheral temperature

DV......... $\frac{V_p - V_s}{2}$ .... one-half of the difference between the central and peripheral velocity

DVP........ $V_p - \bar{V}$ .... numerical difference between the axial fluid velocity and the final uniform velocity

DX(I)...... $\Delta\left(\frac{X}{r_{a_o}}\right)$ .... the change in the non-dimensional ratio DIST(I) between a given cross-section and the previously computed cross-section

ETA + ZETA.. $(\dot{H})_x$ .... numerical value of total enthalpy rate at a given cross-section

FLAG...................... constant used to ensure that the first correct value of CONST is used throughout the program

GAMMA..................... numerical value of Bernoulli's constant for the central fluid

HDOT....... $\frac{\dot{H}}{\pi \rho c_p V_{s_o} r_{a_o}^2}$ .... numerical value of the total enthalpy rate at the entrance to the mixing chamber

PI......... $\frac{(\text{Momentum rate})_\infty}{\pi \rho V_{s_o}^2 r_{a_o}^2}$ .... numerical value of the momentum rate for uniform flow at exit

P(I)....... $\frac{p}{\frac{1}{2}\rho V_{s_o}^2}$ .... non-dimensional ratio of pressure to one-half times the density times the initial secondary fluid velocity squared

PSI(I).... $\dfrac{(\text{Momentum rate})_x}{(\text{Momentum rate})_\infty} = K_m$... numerical multiple of the final momentum rate at any given cross-section

RA(I)...... $\dfrac{r_a}{r_{a_o}} = R_a$.......... non-dimensional ratio of the central radius describing velocity core boundary at a given cross-section to that of the central jet radius

RAT(I)..... $\dfrac{r_{at}}{r_{a_o}} = R_{at}$........ non-dimensional ratio of the central radius describing temperature core boundary at a given cross-section to that of the initial central jet radius

RB(I)...... $\dfrac{r_b}{r_{a_o}} = R_b$.......... non-dimensional ratio of the peripheral radius describing velocity jet boundary at a given cross-section to that of the initial central jet radius

RBT(I)..... $\dfrac{r_{bt}}{r_{a_o}} = R_{bt}$........ non-dimensional ratio of the peripheral radius describing temperature jet boundary at a given cross-section to that of the initial jet radius

RS......... $\dfrac{r_s}{r_{a_o}} = R_s$.......... ratio of peripheral radius of mixing chamber to the initial jet radius

RX & RY..... $\dfrac{r_x}{r_{a_o}}$ & $\dfrac{r_y}{r_{a_o}}$.......... limits of integration for the continuity equation and the momentum equation

RZ......... $\dfrac{r_z}{r_{a_o}}$.............. reference radius during numerical integration of the total enthalpy rate

SIG(J)..... $h\,\Delta\dot{m}$............. incremental numerical segments whose total is ZETA [ZETA = Σ Sig(J)]

TAVE....... $\dfrac{T_p+T_s}{2} = T_{ave}$..... average value of central and peripheral temperature

TBAR....... $\bar{T}$................ final uniform temperature at exit

TAU + OMEGA.. $\dfrac{\left(\text{Momentum rate}\right)_x}{\pi \rho V_{s_o}^2 r_{a_o}^2} = \dot{M}$.. numerical value of the momentum rate at a given cross-section

TAU + SIGMA.. $\dfrac{\dot{m}_x}{\pi \rho V_{s_o} r_{a_o}^2}$...... numerical value of the mass flow rate at a given cross-section

TP(I).......T_p............. numerical value of the central fluid temperature above the initial secondary fluid temperature at the entrance to the mixing chamber

TS(I).......T_s............. numerical value of peripheral fluid temperature above the initial secondary fluid temperature at the entrance to the mixing chamber

VAVE........$\dfrac{V_p + V_s}{2} = V_{ave}$.. average value of the central and peripheral velocity

VBAR........$\overline{V}$............. final uniform velocity at exit

VP(I).......$\dfrac{V_p}{V_{s_o}} = V_p{*}$....... numerical ratio of central fluid velocity to initial secondary fluid velocity at the entrance to the mixing chamber

VS(I).......$\dfrac{V_s}{V_{s_o}} = V_s{*}$....... numerical ratio of peripheral fluid velocity to initial secondary fluid velocity at the entrance to the mixing chamber

1. Introduction

The majority of analyses of the physical properties of fluids in ejectors have used a one-dimensional approach in which uniform properties are assumed at the entrance to the mixing chamber which is of sufficient length for the properties to again become uniform at the exit.[1] For example, consider a primary fluid flowing along the central axis of a cylindrical ejector and a secondary fluid flowing parallel to and completely surrounding it, as shown in Fig. 1. These fluids are separated by a circular boundary. At the entrance to the mixing chamber the boundary between the two fluids is removed and, at this initial cross-section, both the primary and the secondary fluids have uniform but different velocities. As the primary and secondary fluids flow axially through the mixing chamber, the fluids mix and the physical properties of each one of them are affected by the physical properties of the adjacent fluid.[2]

It is the purpose of this investigation, by use of a high speed digital computer and Fortran Programming Language, to predict what the properties in the partially mixed, non-

[1] S. Pai, Fluid Dynamics of Jets, D. Van Nostrand Co., Inc., New York, 1954

[2] R. A. Smith, "Theory and Design of Simple Ejectors", Some Aspects of Fluid Flow, Edward Arnold and Co., London, 1951

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

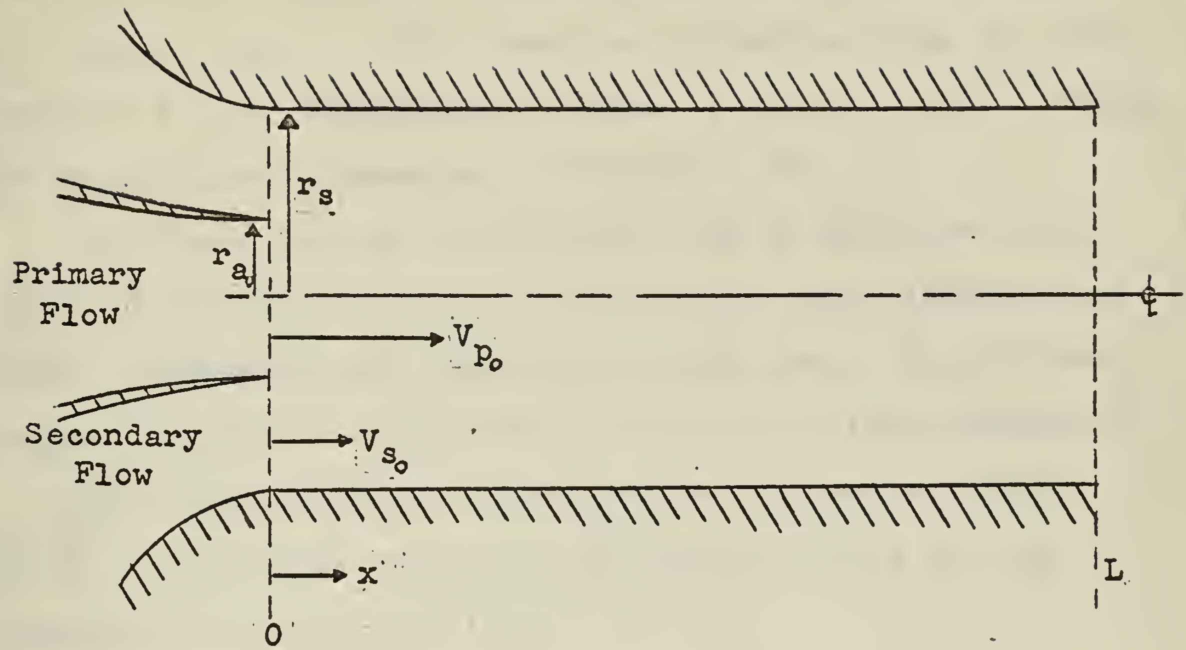


Fig.1 Ejector Geometry and Initial Flow Parameters.

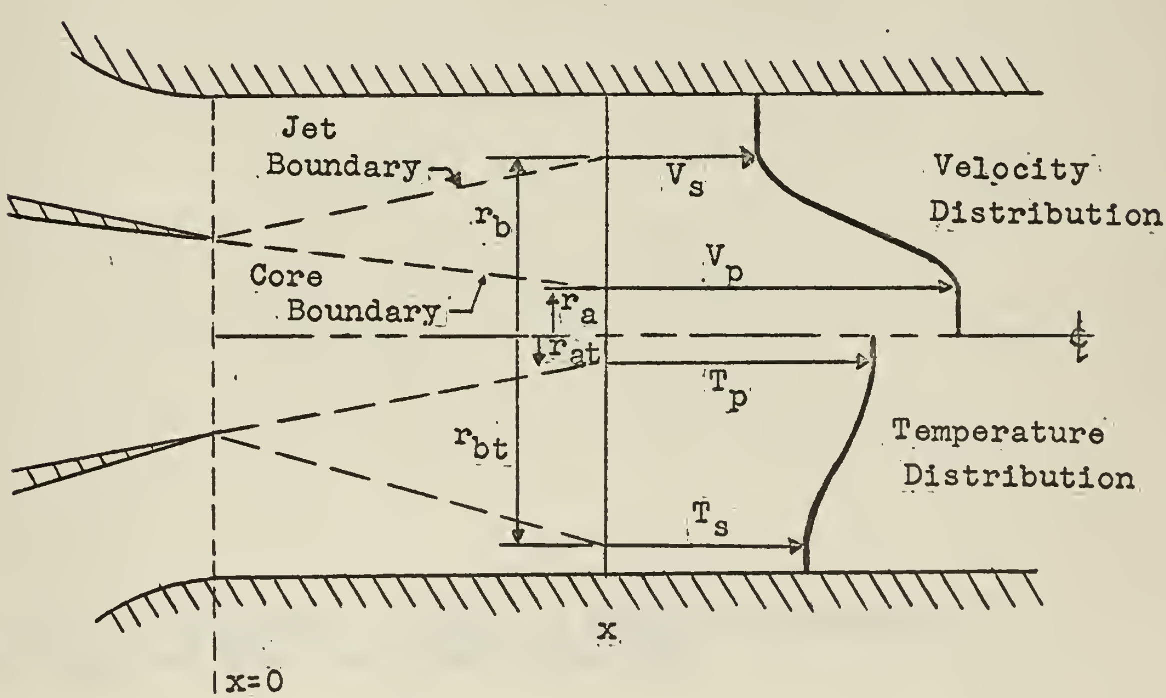


Fig.2 Typical Velocity and Temperature Profiles

uniform sections of the stream will be.[3] Consideration of the fluid flow mechanisms, such as the shear stress within the fluids, leads to the concept of a profile shape for the properties at a given cross-section. A typical profile shape for velocity and temperature is shown in Fig. 2.

The first problem considered is one in which the velocities of the primary and secondary fluids are different but their temperatures and densities are the same. The Fortran Program developed to solve this problem is called EJECTMIX I.

The second problem, considered in the program EJECTMIX II, studies the variations in the velocities and the temperatures of the two fluids.

[3] D. D. McCracken, A Guide to FORTRAN Programming, John Wiley and Sons Inc., New York, 1961

2. Ejector Analysis

In order to analyse the behavior of the physical properties in the mixing chamber of an ejector there are several possible analytic approaches.

The first possible approach is a one-dimensional overall analysis in which uniform properties are assumed at the entrance and uniform properties of the completely mixed fluids are assumed at the exit of the mixing chamber. With this analysis, the region in which mixing of the primary and secondary fluids takes place is not considered. The only regions where the physical properties are studied are at the entrance to the mixing chamber and after the point where uniform flow exists.

A second possible approach is that of an internal analysis which is based on the actual fluid mechanisms. This approach requires the examination of an element of fluid and the forces which are acting upon it. The difficulty here lies in the inability to acquire a usable analytic expression for the shear stress acting upon the element of fluid.

A third approach, the one used in this thesis, is the generation of compatible solutions for the physical properties of the fluids at representative axial cross-sections by means of consideration of Conservation of Mass, Momentum, Bernoulli's Equation and the Energy Equation. In order to apply the above equations to the fluid in the mixing chamber, certain basic assumptions as to the nature of the flow in the

mixing chamber must be made.

First of all, the flow in the mixing chamber is assumed to be divided into regions which are shown in Fig. 3a and Fig. 3c. These regions are defined by the central core boundary and the expanding jet boundary. There are similar velocity and temperature profiles which consist of uniform portions at the central axis and the periphery of the mixing chamber, which are connected by a cosine shaped profile. The assumed profile shapes are shown in Fig. 3b and Fig. 3d. Assuming profile shapes, negligible boundary effects, constant pressure across any given cross-section, and constant temperature until a mixing region is reached, the previously mentioned equations can then be used.

The first equation considered is the Equation of Continuity

$$\dot{m} = \int_0^{r_s} \rho\, V\, dA = \text{Constant}$$

For a typical profile shape, as shown in Fig. 3e, the Equation of Continuity becomes

$$\dot{m} = 2\pi\rho\left(\int_0^{r_a} V_p\, r\, dr + \int_{r_a}^{r_b}\left[V_{ave} + dV\cos\left(\frac{r-r_a}{r_b-r_a}\pi\right)\right] r\, dr + \int_{r_b}^{r_s} V_s\, r\, dr\right)$$

Integrating and evaluating the integral gives

$$\dot{m} = \pi\rho\left(V_p\, r_a^2 + V_{ave}\left[r_b^2 - r_a^2\right] - 4dV\left[\frac{r_b-r_a}{\pi}\right]^2 + V_s\left[r_s^2 - r_b^2\right]\right)$$

Dividing through the entire equation by $\pi\rho V_{s_o} r_{a_o}^2$, the

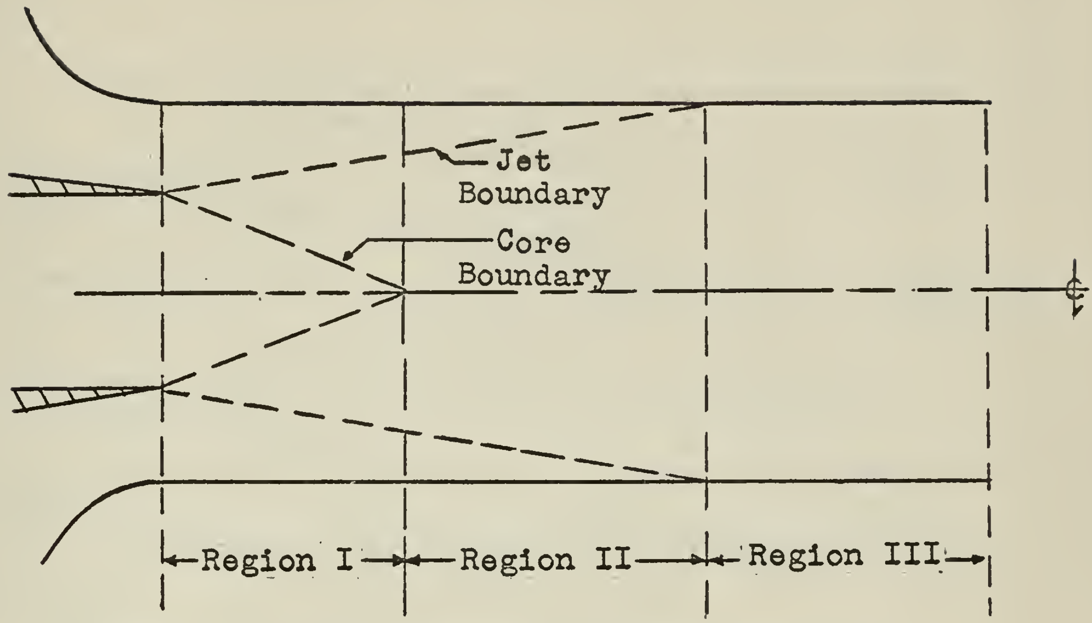


Fig.3a Possible Spread Rate Configuration

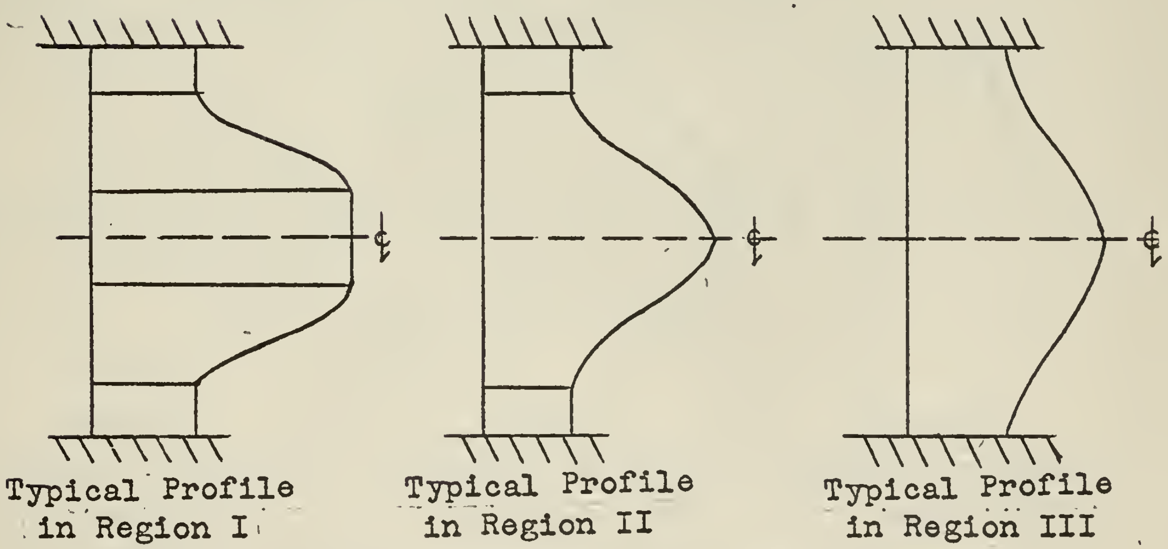


Fig.3b Typical Profiles for Regions of Fig.3a

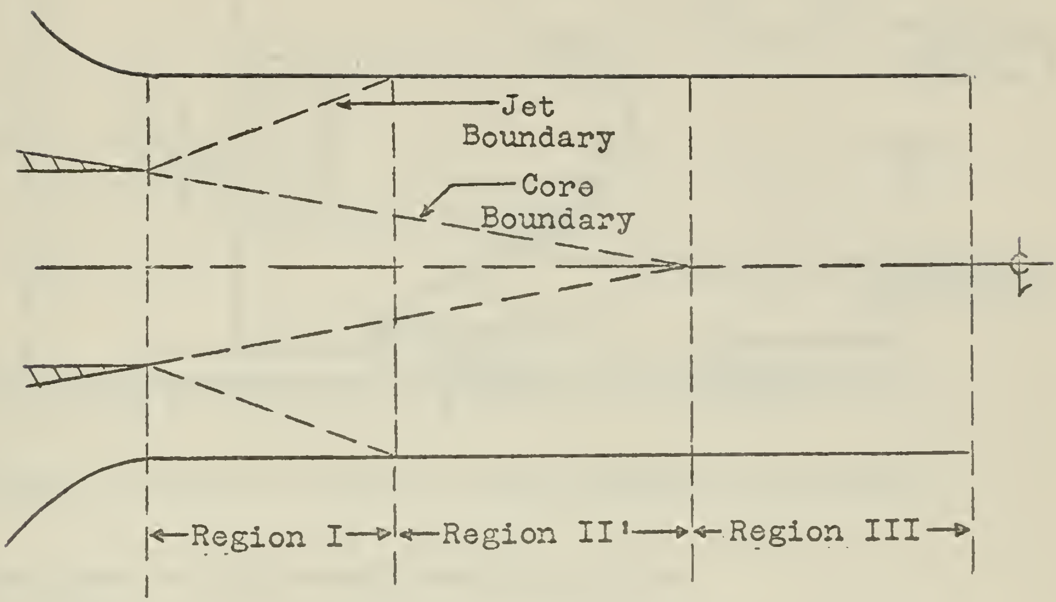
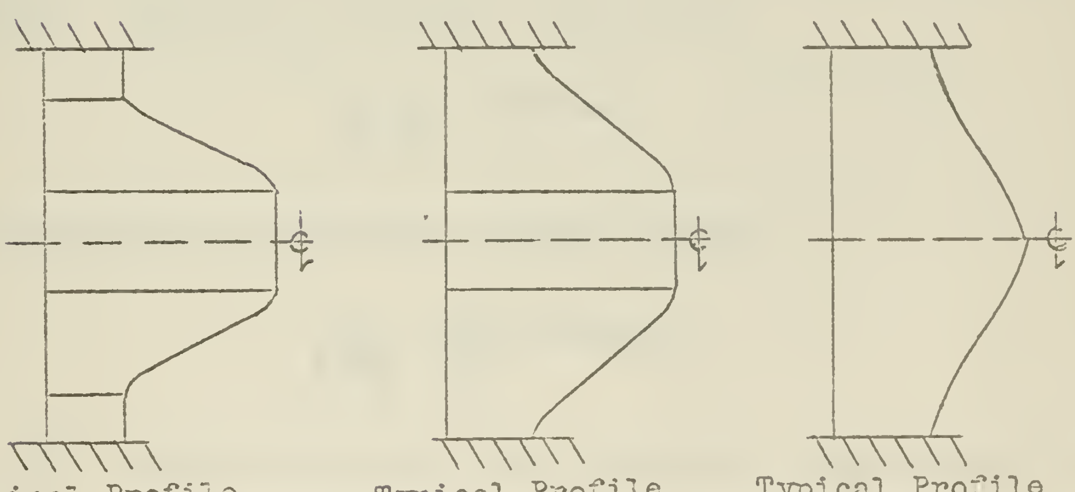


Fig.3c Possible Spread Rate Configuration



Typical Profile in Region I

Typical Profile in Region II'

Typical Profile in Region III

Fig.3d Typical Profiles for Regions of Fig.3c

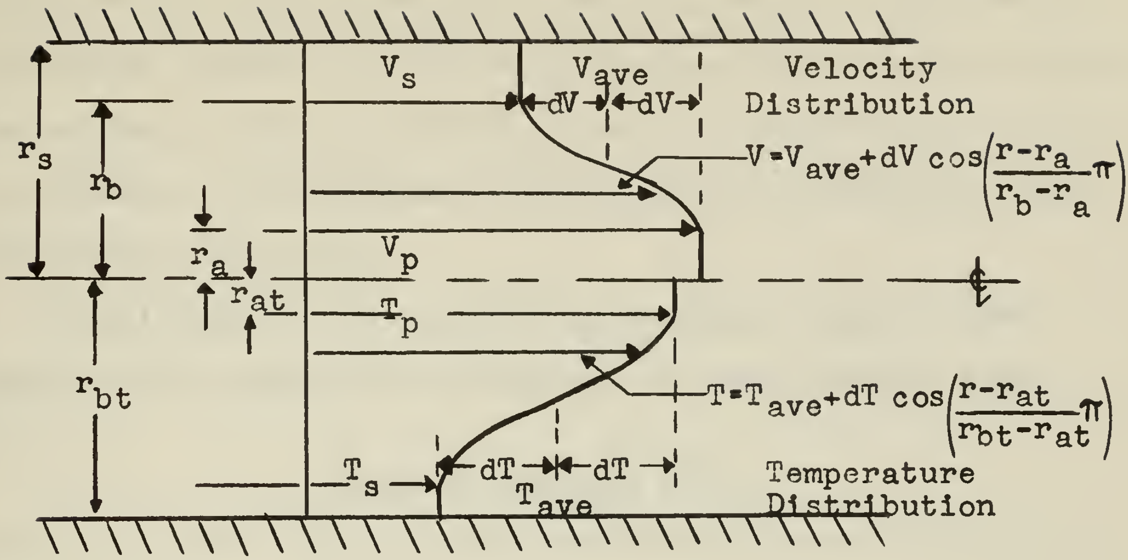


Fig. 3e Typical Profiles with Equation Parameters

equation takes on the non-dimensional form

$$\frac{\dot{m}}{\pi \rho V_{s_o} r_{a_o}^2} = \frac{V_{p_x}}{V_{s_o}} \frac{r_{a_x}^2}{r_{a_o}^2} + \frac{V_{ave_x}}{V_{s_o}} \frac{(r_{b_x}^2 - r_{a_x}^2)}{r_{a_o}^2} - \frac{4 dV_x}{V_{s_o}} \left(\frac{r_{b_x} - r_{a_x}}{\pi r_{a_o}}\right)^2 + \frac{V_{s_x}}{V_{s_o}} \frac{(r_s^2 - r_{b_x}^2)}{r_{a_o}^2}$$

The next consideration is that of Bernoulli's Equation along a streamline in the uniform velocity portions of the profile. Bernoulli's Equation states that

$$\frac{p}{\rho} + \frac{V^2}{2} = \text{Constant}$$

In non-dimensional form the equation becomes

$$\frac{p}{\frac{1}{2}\rho V_{s_o}^2} + \frac{V^2}{V_{s_o}^2} = \text{Constant}$$

The constant associated with Bernoulli's Equation in the primary core is found by evaluating Bernoulli's Equation in

the non-dimensional form along the axis at the entrance to the mixing chamber. Similarly, the constant associated with the secondary cone is determined at the entrance to the mixing chamber. These values will be used in conjunction with the Momentum Equation.

Considering the same profile which was used in the Equation of Continuity and solving for the Momentum rate

$$\text{Momentum rate} = \int \rho V^2 \, dA$$

over the entire cross-section the Momentum Rate Equation becomes

$$\text{Momentum Rate} = 2\pi\rho \left(\int_0^{r_a} V_p^{\,2} \, r \, dr + \int_{r_a}^{r_b} \left[V_{ave} + dV \cos\left(\frac{r - r_a}{r_b - r_a} \pi \right) \right]^2 r \, dr + \int_{r_b}^{r_s} V_s^{\,2} r \, dr \right)$$

Integrating and evaluating the integral

$$\text{Momentum Rate} = \pi\rho \left(V_p^{\,2} \, r_a^{\,2} + V_{ave}\left[r_b^{\,2} - r_a^{\,2} \right] - 8 \, V_{ave} dV \left[\frac{r_b - r_a}{\pi} \right]^2 + dV^2 \left[\pi r_b \frac{(r_b - r_a)}{\pi} - \frac{\pi^2}{2} \left(\frac{r_b - r_a}{\pi} \right)^2 \right] \; V_s^{\,2} \left[r_s^{\,2} - r_b^{\,2} \right] \right)$$

Dividing through the entire equation by $\pi\rho V_{s_o}^{\,2} \, r_{a_o}^{\,2}$ puts the equation in the non-dimensional form

$$\frac{\text{Momentum Rate}}{\pi\rho V_{s_o}^2 r_{a_o}^2} = \frac{V_{p_x}^2}{V_{s_o}^2}\frac{r_{a_x}^2}{r_{a_o}^2} + \frac{V_{ave_x}^2}{V_{s_o}^2}\frac{(r_{b_x}^2 - r_{a_x}^2)}{r_{a_o}^2} - 8\frac{V_{ave_x}\, dV_x}{V_{s_o}^2}\left(\frac{r_{b_x} - r_{a_x}}{\pi r_{a_o}}\right)^2$$

$$\frac{dV_x^2}{V_{s_o}^2}\left(\pi\frac{r_{b_x}}{r_{a_o}^2}(r_{b_x} - r_{a_x}) - \frac{\pi^2}{2}\left[\frac{r_{b_x} - r_{a_x}}{\pi r_{a_o}}\right]^2\right) + \frac{V_{s_x}^2}{V_{s_o}^2}\frac{r_s^2 - r_{b_x}^2}{r_{a_o}^2}$$

The Momentum Equation is then applied to the ejector in the axial direction. Neglecting shear stress at the ejector walls,

$$\Delta p\, \pi r_s^2 = \Delta(\text{Momentum Rate})$$

Defining a non-dimensional Momentum Rate by dividing by $\pi\rho V_{s_o}^2 r_{a_o}^2$, we obtain,

$$\frac{\Delta p}{\rho V_{s_o}^2}\frac{r_s^2}{r_{a_o}^2} = \Delta \dot{M}$$

Defining a non-dimensional pressure difference, Δp^*, as

$$\Delta p^* = \frac{\Delta p}{\frac{1}{2}\rho V_{s_o}^2} = \Delta\dot{M}\,\frac{2r_{a_o}^2}{r_s^2}$$

where $\Delta p = p_x - p_{x=o}$ and $\Delta\dot{M} = \dot{M}_x - \dot{M}_{x=o}$.

The final equation used is the Energy Equation in which the heat transferred to the surroundings is assumed to be zero. The difference in geo-potential energy is negligible and, lastly, the change in kinetic energy is negligible in comparison with the Enthalpy Rate. Thus the Energy Equation becomes

$$\dot{H} = \int d\dot{H} = \int h\, d\dot{m} = 2\pi\rho c_p \int TV\, r dr = \text{Constant}$$

Using the same profile shapes as before, the Enthalpy Rate for a typical axial cross-section becomes

$$\dot{H} = 2\pi\rho c_p \left(\int_0^{r_a} T_p V_p r\,dr + \int_{r_a}^{r_{at}} T_p \left[V_{ave} + dV \cos\left(\frac{r-r_a}{r_b-r_a}\right)\right] r\,dr + \right.$$

$$\int_{r_{at}}^{r_{bt}} \left[T_{ave} + dT \cos\left(\frac{r-r_a}{r_{bt}-r_{at}}\pi\right)\right]\left[V_{ave} + dV \cos\left(\frac{r-r_a}{r_b-r_a}\pi\right)\right] rdr +$$

$$\left. \int_{r_{bt}}^{r_b} T_s \left[V_{ave} + dV \cos\left(\frac{r-r_a}{r_b-r_a}\pi\right)\right] rdr + \int_{r_b}^{r_s} T_s V_s\, rdr \right)$$

The second term of the Enthalpy Rate Equation, namely,

$$2\pi\rho c_p \int_{r_{at}}^{r_{bt}} \left[V_{ave} + dV \cos\left(\frac{r-r_a}{r_b-r_a}\pi\right)\right]\left[T_{ave} + dT \cos\left(\frac{r-r_{at}}{r_{bt}-r_{at}}\pi\right)\right] rdr$$

was solved by numerical integration. The interval between r_{at} and r_{bt} was divided into 100 increments and summed.

The other integrals which make up the total value of the Enthalpy Rate are integrated and evaluated giving

$$\pi\rho c_p \left(V_p T_p r_{at}^2 + V_p T_{ave}\left[r_a^2 - r_{at}^2\right] + \right.$$

$$2V_p\, dT \left[\frac{r_{bt}-r_{at}}{\pi}\right] r_a \operatorname{Sin}\left[\frac{r_a-r_{at}}{r_{bt}-r_{at}}\pi\right] +$$

$$2V_p\, dT \left[\frac{r_{bt}-r_{at}}{\pi}\right]^2 \left[\operatorname{Cos}\left(\frac{r_a-r_{at}}{r_{bt}-r_{at}}\pi\right) - 1\right] +$$

$$V_s T_{ave}\left[r_{bt}^2 - r_b^2\right] - 2V_s\, dT \left[\frac{r_{bt}-r_{at}}{\pi}\right] r_b \operatorname{Sin}\left[\frac{r_b-r_{at}}{r_{bt}-r_{at}}\pi\right] +$$

$$\left. 2V_s\, dT \left[\frac{r_{bt}-r_{at}}{\pi}\right]^2 \left[1 - \operatorname{Cos}\left(\frac{r_b-r_{at}}{r_{bt}-r_{at}}\pi\right)\right] + V_s T_s \left[r_s^2 - r_{bt}^2\right] \right)$$

In order to place this in non-dimensional form, the equation was divided by $\pi \rho c_p V_{s_o} r_{a_o}^2$. Thus the integral for numerical integration becomes

$$\int_{R_{at}}^{R_{bt}} \left[\frac{V_{ave_x}}{V_{s_o}} + \frac{dV_x}{V_{s_o}} \text{Cos}\left(\frac{r - r_{a_x}}{r_{b_x} - r_{a_x}} \pi\right)\right] \left[T_{ave_x} + dT_x \text{ Cos}\left(\frac{r - r_{at_x}}{r_{bt_x} - r_{at_x}} \pi\right)\right] \frac{r}{r_{a_o}} \, d\left[\frac{r}{r_{a_o}}\right]$$

Non-dimensionalizing the evaluated portion of the Enthalpy Rate gives

$$\frac{V_{p_x}}{V_{s_o}} T_{p_x} \frac{r_{at_x}^2}{r_{a_o}^2} + \frac{V_{p_x}}{V_{s_o}} T_{ave_x} \frac{(r_{a_x}^2 - r_{at_x}^2)}{r_{a_o}^2} +$$

$$2\frac{V_{p_x}}{V_{s_o}} dT_x \left(\frac{r_{bt_x} - r_{at_x}}{\pi}\right) \frac{r_{a_x}}{r_{a_o}^2} \text{ Sin}\left(\frac{r_{a_x} - r_{at_x}}{r_{bt_x} - r_{at_x}} \pi\right) +$$

$$2\frac{V_{p_x}}{V_{s_o}} dT_x \left(\frac{r_{bt_x} - r_{at_x}}{\pi \, r_{a_o}}\right)^2 \left(\text{Cos}\left[\frac{r_{a_x} - r_{at_x}}{r_{bt_x} - r_{at_x}} \pi\right] - 1\right) +$$

$$\frac{V_{s_x}}{V_{s_o}} T_{ave_x} \frac{(r_{bt_x}^2 - r_{b_x}^2)}{r_{a_o}^2} - 2\frac{V_{s_x}}{V_{s_o}} dT_x \left(\frac{r_{bt_x} - r_{at_x}}{\pi}\right) \frac{r_{b_x}}{r_{a_o}^2} \text{Sin}\left(\frac{r_{b_x} - r_{at_x}}{r_{bt_x} - r_{at_x}} \pi\right)$$

$$2 \frac{V_{s_x}}{V_{s_o}} dT_x \left(\frac{r_{bt_x} - r_{at_x}}{\pi \, r_{a_o}}\right)^2 \left(1 - \text{Cos}\left[\frac{r_{b_x} - r_{at_x}}{r_{bt_x} - r_{at_x}} \pi\right]\right) + \frac{V_{s_x}}{V_{s_o}} T_{s_x} \frac{(r_s^2 - r_{bt_x}^2)}{r_{a_o}^2}$$

3. Method

The equations employed in EJECTMIX I and EJECTMIX II were used in the following manner. At a given axial position, a core boundary (r_a) is assumed. Primary and secondary velocities (V_p and V_s) are assumed which will be compatible with the Bernoulli Equation for the entrance to the mixing chamber. A jet boundary (r_b) is assumed and the conditions are checked for Conservation of Mass by means of the Continuity Equation. The primary and secondary velocities are corrected by an iterative process until a compatible solution is found.

The velocities found by the Continuity Equation are then used to solve for the pressure at the cross-section under consideration. Pressure changes determined from Bernoulli's Equation along streamlines in the uniform flow velocity regions are compared with the pressure change found from the overall Momentum Rate. If the pressure is found to be incompatible, the jet boundary (r_b) is corrected and once again the Equation of Continuity is used in order to determine compatible central and peripheral velocities. The values found by the Continuity Equation are then used to find a new value of the Momentum Rate, which, in turn, determines a new pressure change which can be used in Bernoulli's Equation to check the velocities found by Continuity. This process repeats itself until the core and jet boundaries (r_a and r_b), the central and peripheral velocities (V_p and V_s) and the pressure change are all compatible in the three basic equa-

[illegible]

[illegible]

[illegible]

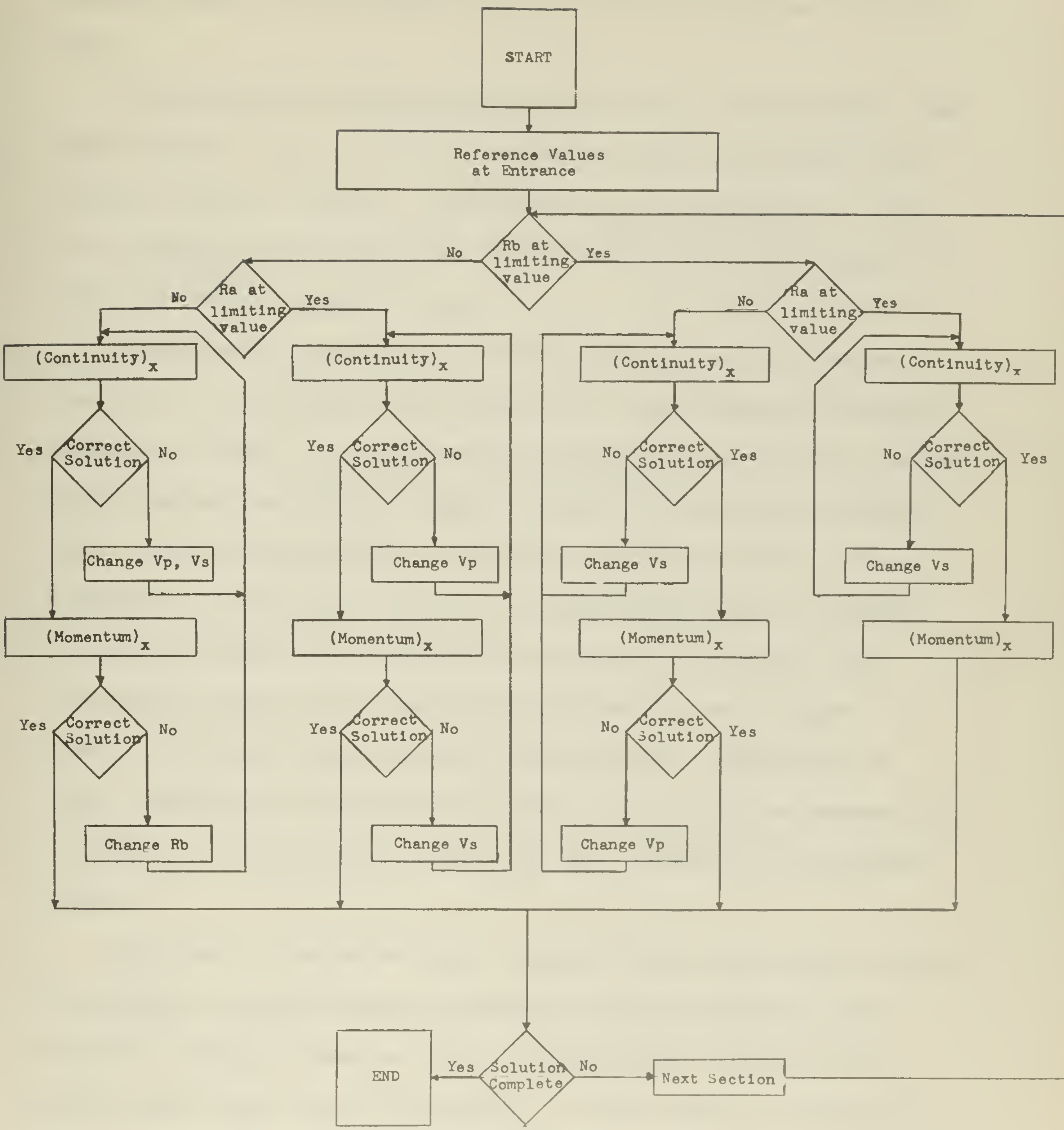


Fig. 4 Block Diagram for EJECTMIX I

tions used. Then the process is repeated for the next cross-section which is displaced axially in the direction of fluid flow.

After several cross-sections have been investigated, the core boundary (r_a) or the jet boundary (r_b) may reach their limiting value, that is, the central core boundary (r_a) may go to zero or the peripheral jet boundary (r_b) may reach the mixing chamber boundary (r_s).

Considering the case of either one of the boundaries reaching its limiting value while the other boundary remains within its limit, the scheme for finding the compatible solution remains exactly the same as that used when both boundaries were within limits, with the exception of the role of Bernoulli's Equation. Since the flow field will no longer be uniform where the boundary has reached its limit, the streamline along which Bernoulli's Equation was assumed to have a constant value will no longer exist. With this in mind, Bernoulli's Equation will only be taken into account in the uniform flow portion which has not reached its limiting value.

When both boundaries have reached their respective limiting values, no additional restrictions are required other than the overall Momentum Rate. Compatible solutions are then found using fixed increments of the central velocity as the criteria for the particular cross-section.

EJECTMIX II is basically the same program as EJECTMIX I

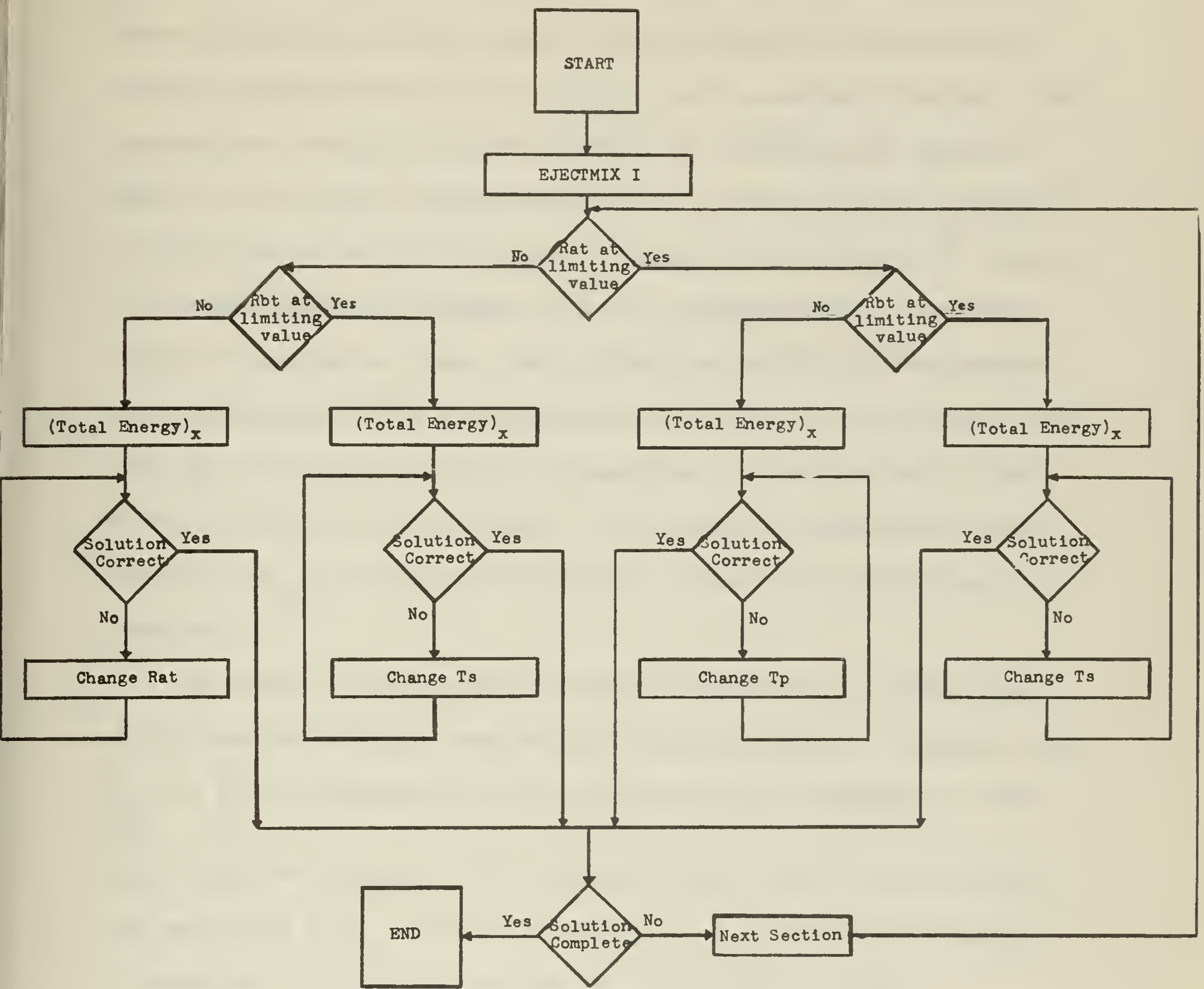


Fig. 5 Block Diagram for EJECTMIX II

with additional restrictions placed upon the flow field. These additional restrictions take the form of variations in the temperatures of the primary and secondary fluids. The temperature boundaries are assumed to spread more rapidly than the velocity region boundaries. This has been verified by experiments on jets of gases flowing in ejectors.[4] Also, the temperature is assumed to be a constant until the temperature boundaries reach their limiting values. The temperature boundaries are assumed and the value of the energy equation at the cross-section in question is compared with that of the initial cross-section. The central temperature core boundary (r_{at}) is corrected until a compatible solution is obtained.

In order to determine the axial position on which each of the representative compatible solutions lie, a spread rate $\left[\frac{d}{dx}(r_b)\right]$ is assumed for the peripheral jet boundary. When this boundary reaches its limiting value, it is assumed that the pressure will continue varying as a smooth curve until it reaches its limiting value in the mixing chamber.

[4] D. Küchemann and J. Weber, Aerodynamics of Propulsion, McGraw-Hill Book Co., Inc., New York, 1953

4. Results

The data which was obtained using EJECTMIX I and EJECTMIX II is presented in graphical form in Appendix 2 and in tabular form in Appendix 3. The properties obtained are presented in groups according to the ratio of the radius of the peripheral boundary to the central jet radius. The values of this given ratio are 1.5, 3.0 and 10.0.

These graphs may be entered with the known physical dimensions of the ejector mixing chamber and the velocity ratio. The ratio of the initial peripheral radius to the central jet radius of ten to one gives an actual area ratio of 100 to one and thus approaches the condition whereby the peripheral fluid actually would be unbounded.

The graphical solutions shown are those in which the fluid was assumed to return to a uniform flow after it had passed down an axial distance equal to ten times the peripheral diameter. The data and graphs are presented as functions of axial distance divided by initial central jet radius (X/r_a). Tabulated data shown in Appendix 3 is the computer output which was used to produce these graphs.

In the course of finding compatible solutions, a temperature difference of 100 degrees was used. By varying only the temperature, the solutions obtained for temperature were directly proportional to each other. With this in mind, the temperature difference for any case can be determined by using a proportionality constant and a 100 degree temperature

difference. Temperatures are measured above a reference temperature which is the secondary fluid temperature at the initial cross-section.

The pressure term as used in Bernoulli's Equation is not significant in its absolute value since it cancels out of the equation when used in these programs. Its significance lies in the change of the non-dimensional pressure term. With each specific case an initial non-dimensional pressure can be determined. This non-dimensional pressure will be a function of the absolute velocities, the pressure and the fluid flowing in the ejector.

Since the initial peripheral velocity ratio is equal to one, a change in the non-dimensional pressure of one, before the velocity jet boundary reaches its limiting value, leads to a zero velocity ratio through the Bernoulli Equation. Any increase in the non-dimensional pressure beyond the value of one leads to compatible solutions in which the peripheral velocity becomes negative. This phenomenon is dependent upon the velocity ratio and the cross-sectional area of the central and peripheral jets.

Negative peripheral velocity ratios first appeared in the range of the values studied at an initial central velocity ratio of ten and an area ratio less than 100 to one. The flow pattern which exists when negative peripheral velocity ratios are generated in the compatible solution is shown in Fig. 6.

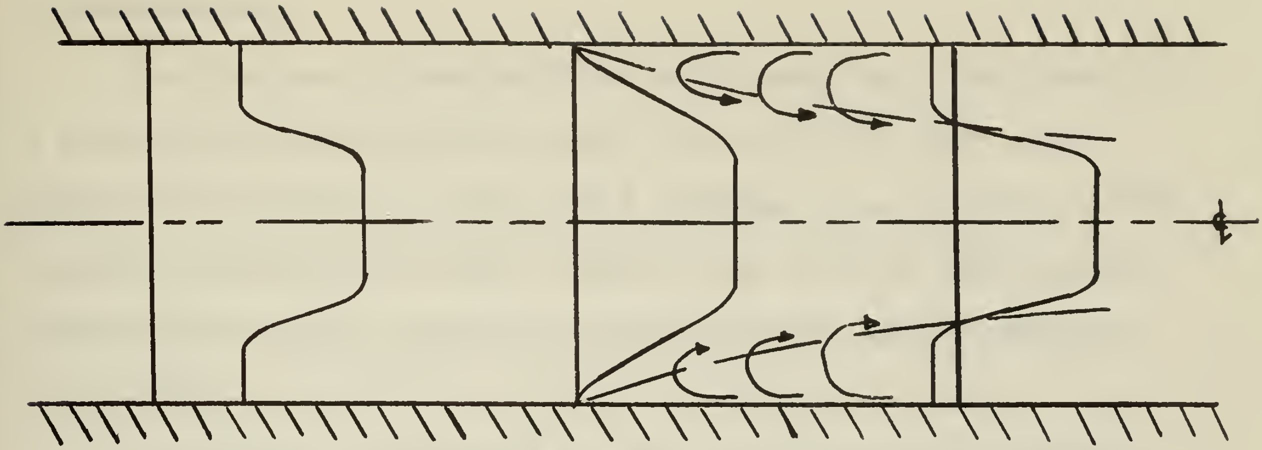


Fig. 6 Profiles and Flow Pattern
with Negative Peripheral Velocities

Throughout most of the program accuracy of one-half of one percent is required in order for the solution to be considered acceptable. This criterion fails during the transition of the peripheral velocity ratio term through its zero values. For this portion of the program accuracy of plus or minus one one-hundredth for the non-dimensional velocity term is required. This change in the criterion ensures that a finite and acceptable error can be established for any value of the peripheral velocity ratio.

The final piece of data presented by the program is the Momentum Factor. This term gives that multiple of the final momentum that exists at any point.

5. Conclusions

The programs presented here represent the first step in a series of programs which would eventually describe the physical properties of any fluid flowing in an ejector. This logical continuation would require that work be done on the compressible fluid problem at both subsonic and supersonic velocities.

The programs contained in this work solve the ejector problem for two incompressible fluids of the same density whose physical properties are initially different. The data obtained is presented here in graphical and tabular form.

BIBLIOGRAPHY

1. S. Pai, Fluid Dynamics of Jets, D. Van Nostrand Co., Inc., New York, 1954

2. R. A. Smith, "Theory and Design of Simple Ejectors", Some Aspects of Fluid Flow, Edward Arnold and Co., London, 1951

3. D. D. McCracken, A Guide to FORTRAN Programming, John Wiley and Sons Inc., New York, 1961

4. D. Küchemann and J. Weber, Aerodynamics of Propulsion, McGraw-Hill Book Co., Inc., New York, 1953

5. G. Rudinger, Wave Diagrams for Non-Steady Flow in Ducts, D. Van Nostrand Co., Inc., New York, 1954

APPENDIX I

Appendix I contains a listing, in FORTRAN language, of the programs EJECTMIX I and EJECTMIX II.

[illegible]

[illegible]

[illegible]

```
      PROGRAM EJECTMIX I
      DIMENSION RA(500),RB(500),VP(500),VS(500),P(500),PSI(500),DP(500)
      READ 101 (VP(1),RS)
  101 FORMAT (2F15.0)
      VS(1) = 1.0
      P(1) = 10.0
      DP(1) = 0.0
      RA(1) = 1.0
      RB(1) = 1.0
      RB(2) = 1.03
      I = 1
      ALPHA = VP(1)*RB(1)**2 + (RS**2 - RB(1)**2)
      BETA = VP(1)**2*RB(1)**2 + (RS**2 - RB(1)**2)
      GAMMA = P(1) + VP(1)**2
      DELTA = P(1) + 1.0
      THETA = GAMMA - DELTA
      PI = ALPHA**2/RS**2
      PSI(1) = BETA/PI
```

```
      WRITE OUTPUT TAPE 4,102
      WRITE OUTPUT TAPE 4,103
      PRINT 133, RA(I),RB(I),DP(I),VP(I),VS(I),PSI(I)
  102 FORMAT (11X,7HCENTRAL,8X,10HPERIPHERAL,7X,8HPRESSURE,8X,
     17HCENTRAL,8X,1CHPERIPHERAL,7X,8HMCMENTUM)
  103 FORMAT (12X,6HRADIUS,10X,6HRADIUS,10X,6HCHANGE,9X,
     18HVELOCITY,8X,8HVELOCITY,9X,6HFACTOR//)
  104 RA(I + 1) = RA(I) - 0.01
      VP(I + 1) = VP(I)
      VS(I + 1) = VS(I)
      I = I + 1
      IF (RS - RB(I)) 122,122,105
  105 IF (RA(I)) 137,106,106
  106 CHI = (RB(I) - RA(I))/3.1415926
  107 TAU = VP(I)*RA(I)**2 + VS(I)*(RS**2 - RB(I)**2)
      RX = RA(I)
      RY =RB(I)
```

```
108 DV = (VP(I) - VS(I))/2.0
    VAVE = VS(I) + DV
    OMEGA = VAVE*(RY**2 - RX**2) -  4.0*DV*CHI**2
    IF (ALPHA - OMEGA - TAU + 0.001) 109,110,110
109 VP(I) = VP(I) - 0.000007
    VS(I) = SQRTF(VP(I)**2 - THETA)
    GO TO 107
110 IF(ALPHA - OMEGA - TAU - 0.001) 112,112,111
111 VP(I) = VP(I) + 0.00001
    VS(I) = SQRTF(VP(I)**2 - THETA)
    GO TO 107
112 TAU = VP(I)**2*RA(I)**2 + VS(I)**2*(RS**2 - RB(I)**2)
    RY = RB(I)
113 SIGMA = VAVE**2*(RY**2 - RX**2) - 8.0*VAVE*DV*CHI**2 +
   1DV**2*CHI*RY*3.1415926 - 4.9348022*CHI**2*DV**2
    DP(I) =(BETA - SIGMA - TAU)/(0.5*RS**2)
    P(I) = P(1) + DP(I)
    IF (1.005*(DELTA - P(I)) - VS(I)**2) 115,136,114
```

[illegible]

```
114 IF (0.995*(DELTA - P(I)) - VS(I)**2) 136,136,116
136 RB(I + 1) = 2.0*RB(I) - RB(I - 1)
    GO TO 132
115 RB(I) = RB(I) + C.COC1
    GO TO 117
116 RB(I) = RB(I) - O.CCC1
117 GO TO 106
122 IF(RA(I)) 147,123,123
123 RB(I) = RS
    RX = RA(I)
    RY = RS
124 DV = (VP(I) - VS(I))/2.0
    TAU = VP(I)*RA(I)**2
    VAVE = VS(I) + DV
    CHI = (RS - RA(I))/3.1415926
    OMEGA = VAVE*(RY**2 - RX**2) -  4.0*DV*CHI**2
    IF (ALPHA - CMECA - TAU + C.001)125,126,126
125  VS(I) = VS(I) - C.CC007
    GC TC 124
```

```
126 IF (ALPHA - OMEGA - TAU - 0.001) 128,128,127
127  VS(I) = VS(I) + 0.0001
    GO TO 124
128 TAU = VP(I)**2*RA(I)**2
129 SIGMA = VAVE**2*(RY**2 - RX**2) - 8.0*VAVE*DV*CHI**2 +
   1DV**2*CHI*RY*3.1415926 - 4.9348022*CHI**2*DV**2
    DP(I) =(BETA - SIGMA - TAU)/(0.5*RS**2)
    P(I) = P(I) + DP(I)
    IF (1.005*(GAMMA - P(I)) - VP(I)**2) 131,132,130
130 IF (0.995*(GAMMA - P(I)) - VP(I)**2) 132,132,135
131 VP(I) = VP(I) - 0.0001
    GO TO 124
132 PSI(I) = (SIGMA +TAU)/PI
    RB(I + 1) = 2.0*RB(I) - RB(I - 1)
    PRINT 133, RA(I),RB(I),DP(I),VP(I),VS(I),PSI(I)
133 FORMAT (5X,6E16.7)
    GO TO 104
135 VP(I) = VP(I) + 0.0001
    GO TO 124
```

```
137 TAU = VS(I)*(RS**2 - RB(I)**2)
    RA(I) = 0.0
    RX = 0.0
    RY = RB(I)
    CHI = RB(I)/3.1415926
138 DV = (VP(I) - VS(I))/2.0
    VAVE = VS(I) + DV
    OMEGA = VAVE*(RY**2 - RX**2) -  4.0*DV*CHI**2
    IF (ALPHA - OMEGA - TAU + 0.001) 140,142,139
139 IF (ALPHA - OMEGA - TAU - 0.001) 142,142,141
140 VP(I) = VP(2) - 0.00007
    GO TO 138
141 VP(I) = VP(I) + 0.0001
    GO TO 138
142 TAU = VS(I)**2*(RS**2 - RB(I)**2)
    SIGMA = VAVE**2*(RY**2 - RX**2) - 8.0*VAVE*DV*CHI**2 +
   1DV**2*CHI*RY*3.1415926 - 4.9348022*CHI**2*DV**2
    DP(I) =(BETA - SIGMA - TAU)/(0.5*RS**2)
```

```
    P(I) = P(1) + DP(I)
    IF (1.005*(DELTA - P(I)) - VS(I)**2) 145,144,143
143 IF (0.995*(DELTA - P(I)) - VS(I)**2) 144,144,146
144 RB(I + 1) = RB(I) + 0.01
    GO TO 132
145  VS(I) = VS(I) - 0.0001
    GO TO 137
146  VS(I) = VS(I) + 0.0001
    GO TO 137
147 RA(I) = 0.0
    RX = 0.0
    RB(I) = RS
    RY = RS
    TAU = 0.0
    CHI = RS/3.1415926
148 DV = (VP(I) - VS(I))/2.0
    VAVE = VS(I) + DV
    OMEGA = VAVE*(RY**2 - RX**2) -  4.0*DV*CHI**2
```

```
      IF (ALPHA - OMEGA + 0.001)150,149,149
  149 IF (ALPHA - OMEGA - 0.001) 152,152,151
  150 VS(I) = VS(I) - 0.00007
      GO TO 148
  151  VS(I) = VS(I) + 0.0001
      GO TO 148
  152 SIGMA = VAVE**2*(RY**2 - RX**2) - 8.0*VAVE*DV*CHI**2 +
     1DV**2*CHI*RY*3.1415926 - 4.9348022*CHI**2*DV**2
      DP(I) =(BETA - SIGMA - TAU)/(0.5*RS**2)
      P(I) = P(1) + DP(I)
      PSI(I) = (SIGMA + TAU)/PI
      PRINT 153, RA(I),RB(I),DP(I),VP(I),VS(I),PSI(I)
  153 FORMAT (5X,6E16.7)
      VP(I + 1) = VP(I) - 0.01
       VS(I + 1) = VS(I) + 0.0001
      IF (VP(I) - VS(I)) 134,154,154
  154 I = I + 1
      GO TO 147
  134 STOP
      END
      END
```

```
      PROGRAM EJECTMIX II
      DIMENSION RA(500),RB(500),VP(500),VS(500),P(500),PSI(500),
     1DP(500),TP(500),TS(500),DIST(500),DX(500),RAT(500),
     2RBT(500),SIG(500)
      READ 101 (VP(1), RS, TP(1))
  101 FORMAT (3F10.0)
      PARAB1 = 0.002
   70 P(1) = 10.0
      DP(1) = 0.0
      TS(1) = 0.0
      VS(1) = 1.0
      RA(1) = 1.0
      RB(1) = 1.0
      RB(2) = 1.1
      RAT(1) = 1.0
      RBT(1) = 1.0
      RBT(0) = 1.0
      DIST(1) = 0.0
```

```
      FLAG = 0.0
      PARAB2 = 1.3*PARAB1
      I = 1
      ALPHA = VP(1)*RB(1)**2 + (RS**2 - RB(1)**2)
      BETA = VP(1)**2*RB(1)**2 + (RS**2 - RB(1)**2)
      GAMMA = P(1) + VP(1)**2
      DELTA = P(1) + 1.0
      THETA = GAMMA - DELTA
      PI = ALPHA**2/RS**2
      PSI(1) = BETA/PI
      HDOT = VP(1)*TP(1)*RA(1)**2
      VBAR = ALPHA/RS**2
      TBAR = HDOT/(VBAR*RS**2)
      WRITE OUTPUT TAPE 4 , 190
  190 FORMAT (1H1)
      WRITE OUTPUT TAPE 4,102
  102 FORMAT (6X,3HX/D,7X,2HRA,8X,2HRP,8X,2HVP,8X,2HVS,8X,3HRAT,
```

```
 17X,3HRBT,6X,2HTP,9X,2HTS,8X,2HDP,5X,8HMOMENTUM//)
  PRINT 133, DIST(I),RA(I),RB(I),VP(I),VS(I),RAT(I),RBT(I),
 1TP(I),TS(I),CP(I),PSI(I)
104 RA(I + 1) = RA(I) - 0.1
    VP(I + 1) = VP(I)
    VS(I + 1) = VS(I)
    I = I + 1
199 IF (RS - RB(I)) 122,122,105
105 IF (RA(I)) 137,106,106
106 CHI = (RB(I) - RA(I))/3.1415926
    DIST(I) = SQRTF((RB(I) - 1.0)/PARAB1)
    DX(I) = DIST(I) - DIST(I - 1)
107 TAU = VP(I)*RA(I)**2 + VS(I)*(RS**2 - RB(I)**2)
    RX = RA(I)
    RY =RB(I)
108 DV = (VP(I) - VS(I))/2.0
    VAVE = VS(I) + DV
    OMEGA = VAVE*(RY**2 - RX**2) -  4.0*DV*CHI**2
    IF (1.005*ALPHA - OMEGA - TAU) 109,110,110
```

```
109 VS(I) = VS(I) + (ALPHA - OMEGA - TAU)/(15.0*RB(I)**2)
    IF (VS(I))193,192,192
192 VP(I) = SQRTF(VS(I)**2 + THETA)
    GO TO 107
    LONG = ABSF(THETA - VS**2)
193 VP(I) = SQRTF(LONG)
    GO TO 107
110 IF (0.995*ALPHA - OMEGA - TAU) 112,112,109
112 TAU = VP(I)**2*RA(I)**2 + VS(I)**2*(RS**2 - RB(I)**2)
    RY = RB(I)
113 SIGMA = VAVE**2*(RY**2 - RX**2) - 8.0*VAVE*DV*CHI**2 +
   1DV**2*CHI*RY*3.1415926 - 4.9348022*CHI**2*DV**2
    DP(I) =(BETA - SIGMA - TAU)/(0.5*RS**2)
    P(I) = P(1) + DP(I)
    IF (1.005*(GAMMA - P(I)) - VP(I)**2) 115,136,114
114 IF (0.995*(GAMMA - P(I)) - VP(I)**2) 136,136,116
136 RB(I + 1) = 2.0*RB(I) - RB(I - 1)
    GO TO 132
```

```
115 RB(I) = RB(I) + 0.001
    GO TO 199
116 RB(I) = RB(I) - 0.001
117 GO TO 199
122 IF(RA(I)) 147,123,123
123 RB(I) = RS
    RX = RA(I)
    RY = RS
    DX(I) = DX(I - 1)*(RA(I)-RA(I-1))/(RA(I-1)-RA(I-2))
    DIST(I) = DIST(I-1) + DX(I)
124 DV = (VP(I) - VS(I))/2.0
    TAU = VP(I)*RA(I)**2
    VAVE = VS(I) + DV
    CHI = (RS - RA(I))/3.1415926
    OMEGA = VAVE*(RY**2 - RX**2) -  4.0*DV*CHI**2
    IF (1.005*ALPHA - OMEGA - TAU) 125,126,126
125 VS(I) = VS(I) + (ALPHA - OMEGA - TAU)/(50.0*RS**2)
    GO TO 124
126 IF (0.995*ALPHA - OMEGA - TAU) 128,128,125
```

```
128 TAU = VP(I)**2*RA(I)**2
129 SIGMA = VAVE**2*(RY**2 - RX**2) - 8.0*VAVE*DV*CHI**2 +
   1DV**2*CHI*RY*3.1415926 - 4.9348022*CHI**2*DV**2
    DP(I) =(BETA - SIGMA - TAU)/(0.5*RS**2)
    P(I) = P(1) + DP(I)
    IF (1.005*(GAMMA - P(I)) - VP(I)**2) 131,132,130
130 IF (0.995*(GAMMA - P(I)) - VP(I)**2) 132,132,135
131 VP(I) = VP(I) - 0.0001
    GO TO 124
132 PSI(I) = (SIGMA +TAU)/PI
    RAT(I) = RA(I) - 0.01*DIST(I)
    IF (RAT(I)) 313,300,300
300 TP(I) = TP(1)
    RBT(I) = 1.0 + PARAB2*DIST(I)**2
301 IF (RS - RBT(I)) 307,93,93
 93 TS(I) = TS(1)
302 ZETA = 0.0
    DA = (RB(I) - RA(I))*3.1415926/(100.0*(RBT(I) - RAT(I)))
```

```
      DB = (RB(I) - RA(I))*3.1415926/(100.0*(RB(I) - RA(I)))
      DT = (TP(I) - TS(I))/2.0
      RZ = RA(I) + 0.005*(RB(I) - RA(I))
      CHIA = (RAT(I) - RA(I))/3.1415926
      ANA = (RA(I) - RAT(I))*3.1415926/(RBT(I) - RAT(I))
      CHIB = (RBT(I) - RAT(I))/3.1415926
      ANB = (RB(I) - RAT(I))*3.1415926/(RBT(I) -  RAT(I))
      ANC = 0.0
      TAVE = 0.5*(TP(I) + TS(I))
      ETA = VP(I)*TP(I)*RAT(I)**2+VP(I)*TAVE*(RA(I)**2-RAT(I)**2)+
     12.0*VP(I)*DT*CHIB*RA(I)*SINF(ANA)+2.0*VP(I)*DT*CHIB**2*
     2(COSF(ANA)-1.0)+VS(I)*TAVE*(RBT(I)**2-RB(I)**2)-
     32.0*VS(I)*DT*CHIB*RB(I)*SINF(ANB)+2.0*VS(I)*DT*CHIB**2*
     4(1.0-COSF(ANB))+VS(I)*TS(I)*(RS**2-RBT(I)**2)
      DO 303 J = 1,100
      SIG(J) = (VAVE+DV*COSF(ANC))*(TAVE+DT*COSF(ANA))*
     1((RZ+0.01*(RB(I)-RA(I)))**2-RZ**2)
      ANA = ANA + DA
      ANC = ANC + DB
```

```
      RZ = RZ + 0.01*(RB(I) - RA(I))
      ZETA = ZETA + SIG(J)
  303 CONTINUE
      IF (1.005*HDOT - ETA - ZETA) 305,328,304
  304 IF (0.995*HDOT - ETA - ZETA) 328,328,305
  305 RAT(I) = RAT(I) + (HDOT-ETA-ZETA)/(100.0*TP(I))
      IF (RAT(I)) 170,302,302
  170 RAT(I) = 0.C
      GC TC 316
  307 RBT(I) = RS
      RAT(I) = RAT(I-1)**2/RAT(I-2)
      IF (RAT(I)) 321,90,90
   90 DA = (RS    - RA(I))*3.1415926/(100.0*(RBT(I) - RAT(I)))
      DB = (RS    - RA(I))*3.1415926/(100.0*(RB(I) - RA(I)))
      TS(I) = TS(I - 1)**2/TS(I - 2)
  308 DT = (TP(I) - TS(I))/2.0
   83 ZETA = 0.0
      RZ = RA(I) + 0.005*(RB(I) - RA(I))
```

```
      CHIA = (RAT(I) - RA(I))/3.1415926
      ANA = (RA(I) - RAT(I))*3.1415926/(RBT(I) - RAT(I))
      CHIB = (RBT(I) - RAT(I))/3.1415926
      ANB = (RB(I) - RAT(I))*3.1415926/(RBT(I) -  RAT(I))
      ANC = 0.0
      TAVE = 0.5*(TP(I) + TS(I))
      ETA = VP(I)*TP(I)*RAT(I)**2+VP(I)*TAVE*(RA(I)**2-RAT(I)**2)+
     12.0*VP(I)*DT*CHIB*RA(I)*SINF(ANA)+2.0*VP(I)*DT*CHIB**2*
     2(COSF(ANA)-1.0)+VS(I)*TAVE*(RBT(I)**2-RB(I)**2)-
     32.0*VS(I)*DT*CHIB*RB(I)*SINF(ANB)+2.0*VS(I)*DT*CHIB**2*
     4(1.0-COSF(ANB))+VS(I)*TS(I)*(RS**2-RBT(I)**2)
      DO 309 J = 1,100
      SIG(J) = (VAVE+DV*COSF(ANC))*(TAVE+DT*COSF(ANA))*
     1((RZ+0.01*(RB(I)-RA(I)))**2-RZ**2)
      ANA = ANA + DA
      ANC = ANC + DP
      RZ = RZ + 0.01*(RB(I) - RA(I))
      ZETA = ZETA + SIG(J)
  309 CONTINUE
```

```
      IF (1.005*HDCT - ETA - ZETA)  311,328,310
  310 IF (0.995*HDCT - ETA - ZETA)  328,328,311
  311 TS(I) = TS(I) + (HDOT - ETA - ZETA)/(20.0*RS**2)
      GO TO 308
  313 RBT(I) = 1.0 + PARAB2*DIST(I)**2
      TP(I) = TP(I - 1)**2/TP(I - 2)
  314 IF (RS - RBT(I)) 321,321,315
  315 TS(I) = TS(1)
      RAT(I) = 0.0
  316 ZETA = 0.0
      DA = (RB(I) - RA(I))*3.1415926/(100.0* RBT(I))
      DB = (RB(I) - RA(I))*3.1415926/(100.0*(RB(I) - RA(I)))
      DT = (TP(I) - TS(I))/2.0
      RZ = RA(I) + 0.005*(RB(I) - RA(I))
      CHIA = (RAT(I) - RA(I))/3.1415926
      ANA = (RA(I) - RAT(I))*3.1415926/(RBT(I) - RAT(I))
      CHIB = (RBT(I) - RAT(I))/3.1415926
      ANB = (RB(I) - RAT(I))*3.1415926/(RBT(I) -  RAT(I))
```

```
      ANC = 0.0
      TAVE = 0.5*(TP(I) + TS(I))
      ETA = VP(I)*TP(I)*RAT(I)**2+VP(I)*TAVE*(RA(I)**2-RAT(I)**2)+
     12.0*VP(I)*DT*CHIB*RA(I)* SINF(ANA)+2.0*VP(I)*DT*CHIB**2*
     2(COSF(ANA)-1.0)+VS(I)*TAVE*(RBT(I)**2-RB(I)**2)-
     32.0*VS(I)*DT*CHIB*RB(I)*SINF(ANB)+2.0*VS(I)*DT*CHIB**2*
     4(1.0-COSF(ANB))+VS(I)*TS(I)*(RS**2-RBT(I)**2)
      DO  317 J = 1,100
      SIG(J) = (VAVE+DV*COSF(ANC))*(TAVE+DT*COSF(ANA))*
     1((RZ+0.01*(RB(I)-RA(I)))**2-RZ**2)
      ANA = ANA + DA
      ANC = ANC + DB
      RZ = RZ + 0.01*(RB(I) - RA(I))
      ZETA = ZETA + SIG(J)
  317 CONTINUE
      IF (1.005 *HDOT - ETA - ZETA ) 319,328,318
  318 IF (0.995*HDOT - ETA - ZETA )  328,328,319
  319 TP(I) = TP(I) +(HDOT - ETA - ZETA)/((RA(I) + 0.5*(RBT(I) -
     1RAT(I)))**2 *VP(I)*5.0)
```

```
      IF (TP(I) - TP(1)) 316,316,320
  320 TP(I) = TP(1)
      GO TO 302
  321 RAT(I) = 0.0
      RBT(I) = RS
      TS(I) = 2.0*TS(I - 1) - TS(I - 2)
      DVP = VP(I) - VBAR
      DA = (RB(I) - RA(I))*3.1415926/(100.0*RS)
      DB = (RB(I) - RA(I))*3.1415926/(100.0*(RB(I) - RA(I)))
      IF (FLAG) 322,322,378
  322 STAR = (TP(I-1)-TBAR)*VBAR/((VP(I-1)-VBAR)*TBAR)
      TP(I) = TBAR + STAR*DVP*TBAR/VBAR
      CONST = (TP(I) - TBAR)*VBAR/((VP(I) - VBAR)*TBAR)
      GO TO 323
  378 TP(I) = TBAR + CONST*DVP*TBAR/VBAR
  323 ZETA = 0.0
  365 DT = (TP(I) - TS(I))/2.0
      RZ = RA(I) + 0.005*(RB(I) - RA(I))
```

```
      CHIA = (RAT(I) - RA(I))/3.1415926
      ANA = (RA(I) - RAT(I))*3.1415926/(RBT(I) - RAT(I))
      CHIB = (RBT(I) - RAT(I))/3.1415926
      ANB = (RB(I) - RAT(I))*3.1415926/(RBT(I) - RAT(I))
      ANC = 0.0
      TAVE = 0.5*(TP(I) + TS(I))
      ETA = VP(I)*TP(I)*RAT(I)**2+VP(I)*TAVE*(RA(I)**2-RAT(I)**2)+
     12.0*VP(I)*DT*CHIB*RA(I)*SINF(ANA)+2.0*VP(I)*DT*CHIB**2*
     2(COSF(ANA)-1.0)+VS(I)*TAVE*(RBT(I)**2-RB(I)**2)-
     32.0*VS(I)*DT*CHIB*RB(I)*SINF(ANB)+2.0*VS(I)*DT*CHIB**2*
     4(1.0-COSF(ANB))+VS(I)*TS(I)*(RS**2-RBT(I)**2)
      DO 324  J = 1,100
      SIG(J) = (VAVE+DV*COSF(ANC))*(TAVE+DT*COSF(ANA))*
     1((RZ+0.01*(RB(I)-RA(I)))**2-RZ**2)
      ANA = ANA + CA
      ANC = ANC + CB
      RZ = RZ + 0.01*(RB(I) - RA(I))
      ZETA = ZETA + SIG(J)
  324 CONTINUE
```

```
      IF (1.005*HDCT - ETA - ZETA) 326,366,325
  325 IF (0.995*HDCT - ETA - ZETA) 366,366,326
  326 TS(I) = TS(I) + (HDOT - ETA - ZETA)/(0.5*(VP(I) + VS(I))*RS**2)
      GO TO 323
  366 FLAG = FLAG + 1.0
  328 RB(I + 1) = 2.0*RB(I) - RB(I - 1)
      PRINT 133, DIST(I),RA(I),RB(I),VP(I),VS(I),RAT(I),RBT(I),
     1TP(I),TS(I),DP(I),PSI(I)
  133 FORMAT (11F10.4/)
      GO TO 104
  135 VP(I) = VP(I) + 0.0001
      GO TO 124
  137 TAU = VS(I)*(RS**2 - RB(I)**2)
      RA(I) = 0.0
      RX = 0.0
      RY = RB(I)
      CHI = RB(I)/3.1415926
      DIST(I) = SQRTF((RB(I) - 1.0)/PARAB1)
```

```
      DX(I) = DIST(I) - DIST(I - 1)
  138 DV = (VP(I) - VS(I))/2.0
      VAVE = VS(I) + DV
      OMEGA = VAVE*(RY**2 - RX**2) -  4.0*DV*CHI**2
      IF (1.005*ALPHA - OMEGA - TAU) 140,142,139
  139 IF (0.995*ALPHA - OMEGA - TAU) 142,142,140
  140 VP(I) = VP(I) + (ALPHA - OMEGA - TAU)/(10.0*RS**2)
      GO TO 138
  142 TAU = VS(I)**2*(RS**2 - RB(I)**2)
      SIGMA = VAVE**2*(RY**2 - RX**2) - 8.0*VAVE*DV*CHI**2 +
     1DV**2*CHI*RY*3.1415926 - 4.9348022*CHI**2*DV**2
      DP(I) =(BETA - SIGMA - TAU)/(0.5*RS**2)
      P(I) = P(1) + DP(I)
   50 IF (VS(I) - 0.2) 51,196,196
   51 VEL = ABSF (CELTA - P(I))
      IF (DELTA - P(I)) 52,53,53
   52 VSB = -SQRTF(VEL)
      GO TO 54
   53 VSB = SQRTF(VEL)
```

```
 54 IF (VSB - VS(I) + 0.01)145,55,55
 55 IF (VSB - VS(I) - 0.01)144,144,146
196 IF (1.005*(DELTA - P(I)) - VS(I)**2) 145,144,143
143 IF (0.995*(DELTA - P(I)) - VS(I)**2) 144,144,146
144 RB(I + 1) = RB(I) + 0.01
    GO TO 132
145  VS(I) = VS(I) - 0.001
    GO TO 137
146 VS(I) = VS(I) + 0.009
    GO TO 137
147 RA(I) = 0.0
    RX = 0.0
    RB(I) = RS
    RY = RS
    TAU = 0.0
    CHI = RS/3.1415926
    VP(I) = VP(I - 1) - 2.0
148 DV = (VP(I) - VS(I))/2.0
```

```
      VAVE = VS(I) + DV
      OMEGA = VAVE*(RY**2 - RX**2) -  4.0*DV*CHI**2
      IF (ALPHA - CMEGA + 0.001)150,149,149
  149 IF (ALPHA - CMEGA - 0.001) 152,152,151
  150 VS(I) = VS(I) - 0.00007
      GO TO 148
  151  VS(I) = VS(I) + 0.0001
      GO TC 148
  152 SIGMA = VAVE**2*(RY**2 - RX**2) - 8.0*VAVE*DV*CHI**2 +
     1DV**2*CHI*RY*3.1415926 - 4.9348022*CHI**2*DV**2
      DP(I) =(BETA - SIGMA - TAU)/(0.5*RS**2)
      P(I) = P(1) + DP(I)
      DX(I) = DP(I)*DP(I - 2)*DX(I - 1)**2/(DX(I - 2)*DP(I - 1)**2)
      DIST(I) = DIST(I - 1) + DX(I)
      PSI(I) = (SIGMA + TAU)/PI
  371 RAT(I) = 0.0
      RBT(I) = RS
      TS(I) = TS(I - 1)**2/TS(I - 2)
      DVP = VP(I) - VBAR
```

```
      DA = (RB(I) - RA(I))*3.1415926/(100.0*RS)
      DB = (RB(I) - RA(I))*3.1415926/(100.0*(RB(I) - RA(I)))
      IF (FLAG) 372,372,379
  372 STAR = (TP(I-1)-TBAR)*VBAR/((VP(I-1)-VBAR)*TBAR)
      TP(I) = TBAR + STAR*DVP*TBAR/VBAR
      CONST = (TP(I) - TBAR)*VBAR/((VP(I) - VBAR)*TBAR)
      GO TO 373
  379 TP(I) = TBAR + CONST*DVP*TBAR/VBAR
  373 ZETA = 0.0
  360 DT = (TP(I) - TS(I))/2.0
      RZ = RA(I) + 0.005*(RB(I) - RA(I))
      CHIA = (RAT(I) - RA(I))/3.1415926
      ANA = (RA(I) - RAT(I))*3.1415926/(RBT(I) - RAT(I))
      CHIB = (RBT(I) - RAT(I))/3.1415926
      ANB = (RB(I) - RAT(I))*3.1415926/(RBT(I) -  RAT(I))
      ANC = 0.0
      TAVE = 0.5*(TP(I) + TS(I))
      ETA = VP(I)*TP(I)*RAT(I)**2+VP(I)*TAVE*(RA(I)**2-RAT(I)**2)+
```

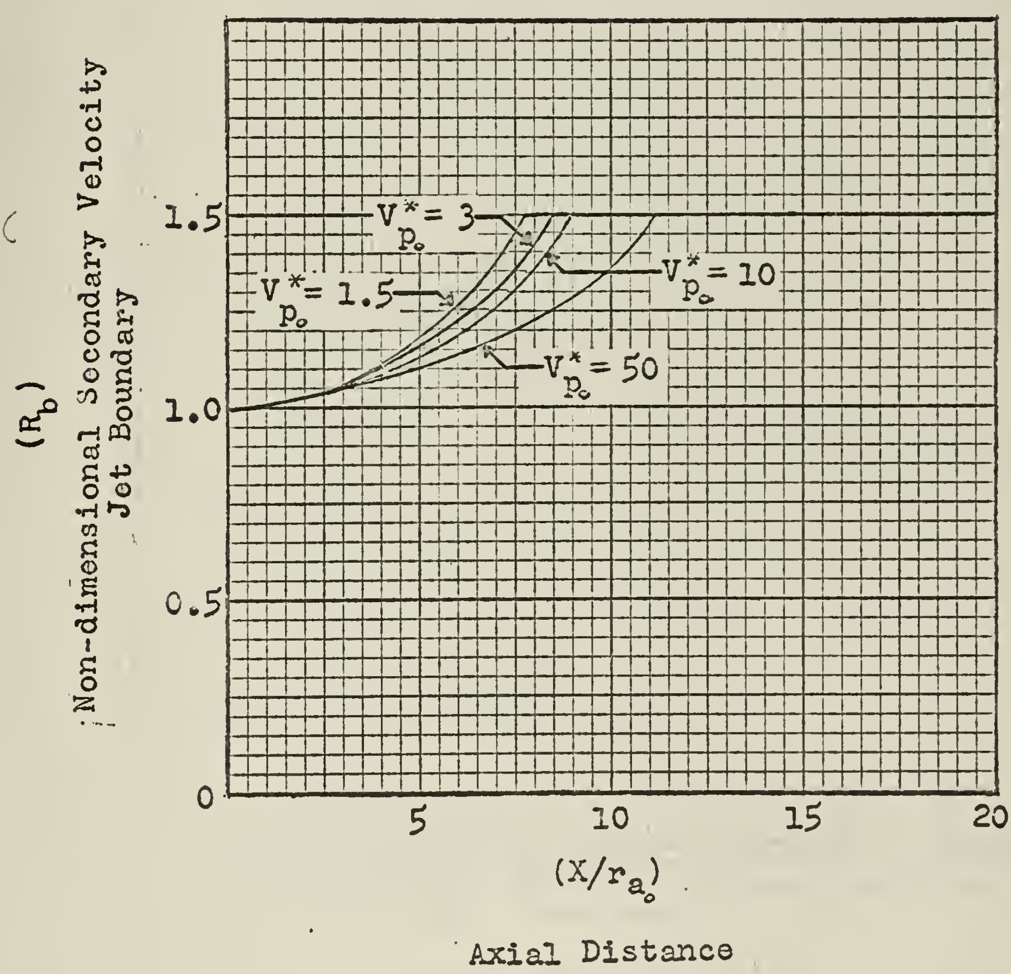
```
     12.0*VP(I)*DT*CHIB*RA(I)*SINF(ANA)+2.0*VP(I)*DT*CHIB**2*
     2(COSF(ANA)-1.0)+VS(I)*TAVE*(RBT(I)**2-RB(I)**2)-
     32.0*VS(I)*DT*CHIB*RB(I)*SINF(ANB)+2.0*VS(I)*DT*CHIB**2*
     4(1.0-COSF(ANB))+VS(I)*TS(I)*(RS**2-RBT(I)**2)
      DO 374  J = 1,100
      SIG(J) = (VAVE+DV*COSF(ANC))*(TAVE+CT*COSF(ANA))*
     1((RZ+0.01*(RB(I)-RA(I)))**2-RZ**2)
      ANA = ANA + DA
      ANC = ANC + DB
      RZ = RZ + 0.01*(RB(I) - RA(I))
      ZETA = ZETA + SIG(J)
  374 CONTINUE
      IF (1.005*HDCT - ETA - ZETA) 376,350,375
  375 IF (0.995*HDCT - ETA - ZETA) 350,350,376
  376 TS(I) = TS(I) + (HDOT - ETA - ZETA)/(0.5*(VP(I) + VS(I))*RS**2)
      GO TO 373
  350 FLAG = FLAG + 1.0
      IF (VP(I) - VS(I)) 400,401,401
```

```
400 VP(I) = VAVE
    VS(I) = VAVE + 0.000000001
    TP(I) = TAVE
    TS(I) = TAVE
    PSI(I) = 1.0
401 PRINT 153, DIST(I),RA(I),RB(I),VP(I),VS(I),RAT(I),RBT(I),
   1TP(I),TS(I),DP(I),PSI(I)
153 FORMAT (11F1C.4/)
    VP(I + 1) = VP(I) - 2.0
     VS(I + 1) = VS(I) + 0.0001
    IF (VP(I) - VS(I)) 134,154,154
154 I = I + 1
    GO TO 147
134 WRITE OUTPUT TAPE 4 , 191
191 FORMAT (1H1)
 71 STOP
    END
    END
```

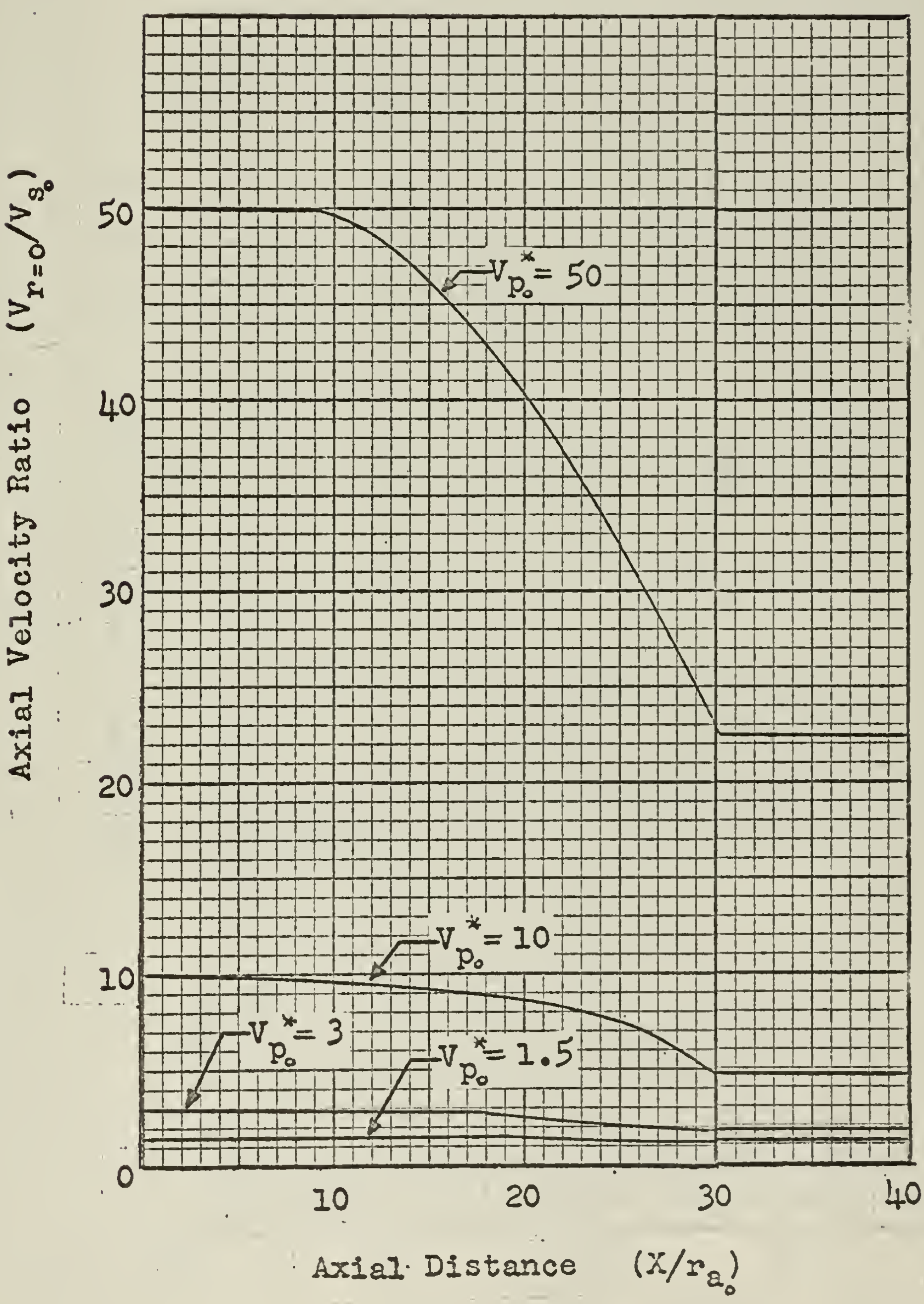
APPENDIX II

Appendix II is a graphical presentation of the data obtained by programs EJECTMIX I and EJECTMIX II. Various physical characteristics are plotted against the axial distance. The axial distance is expressed as a multiple of the central jet radius. The various curves shown represent data obtained using different initial central velocity ratios.

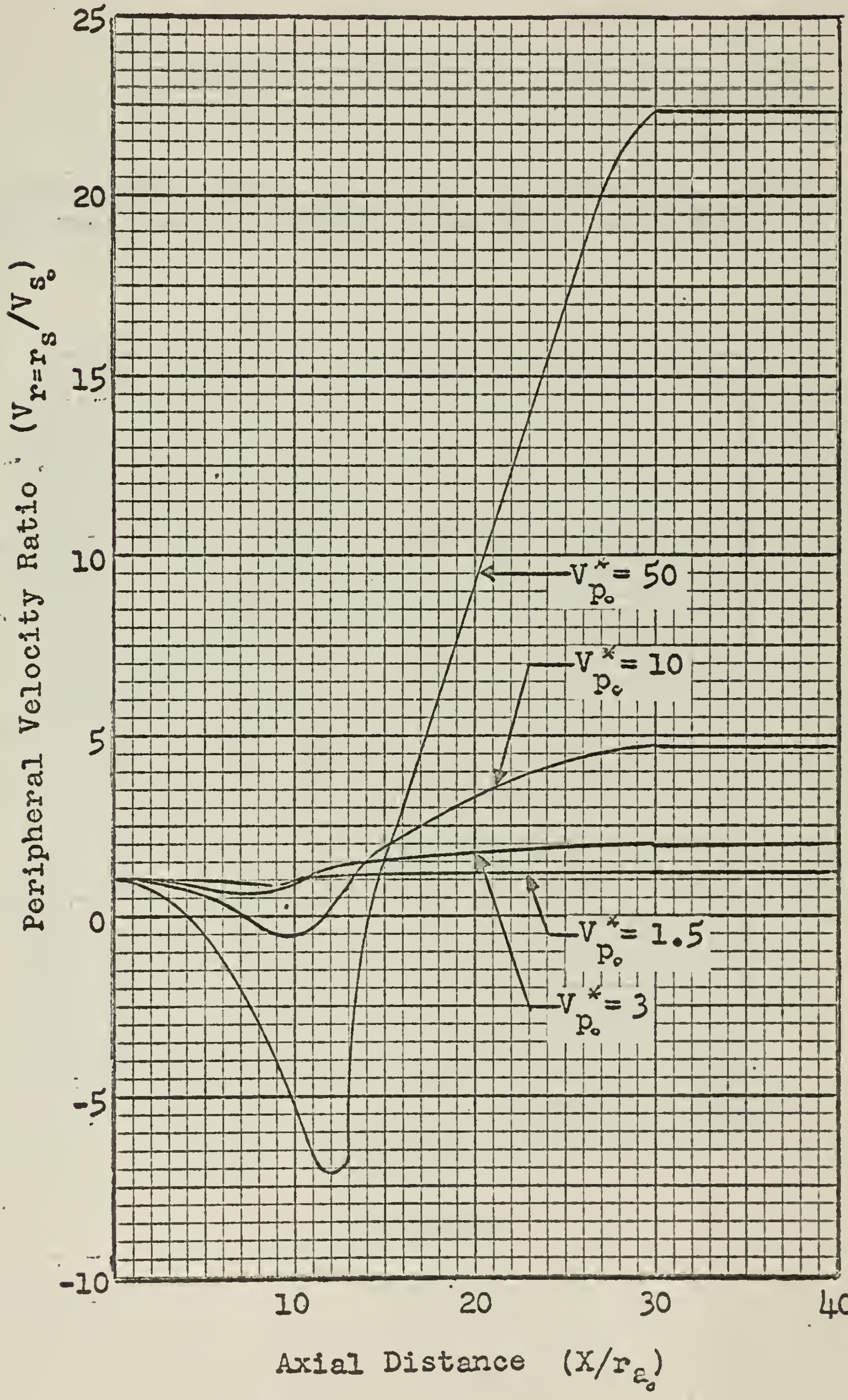
Non-dimensional Secondary Velocity Jet Boundary vs. Axial Distance for an Area Ratio of 2.25:1



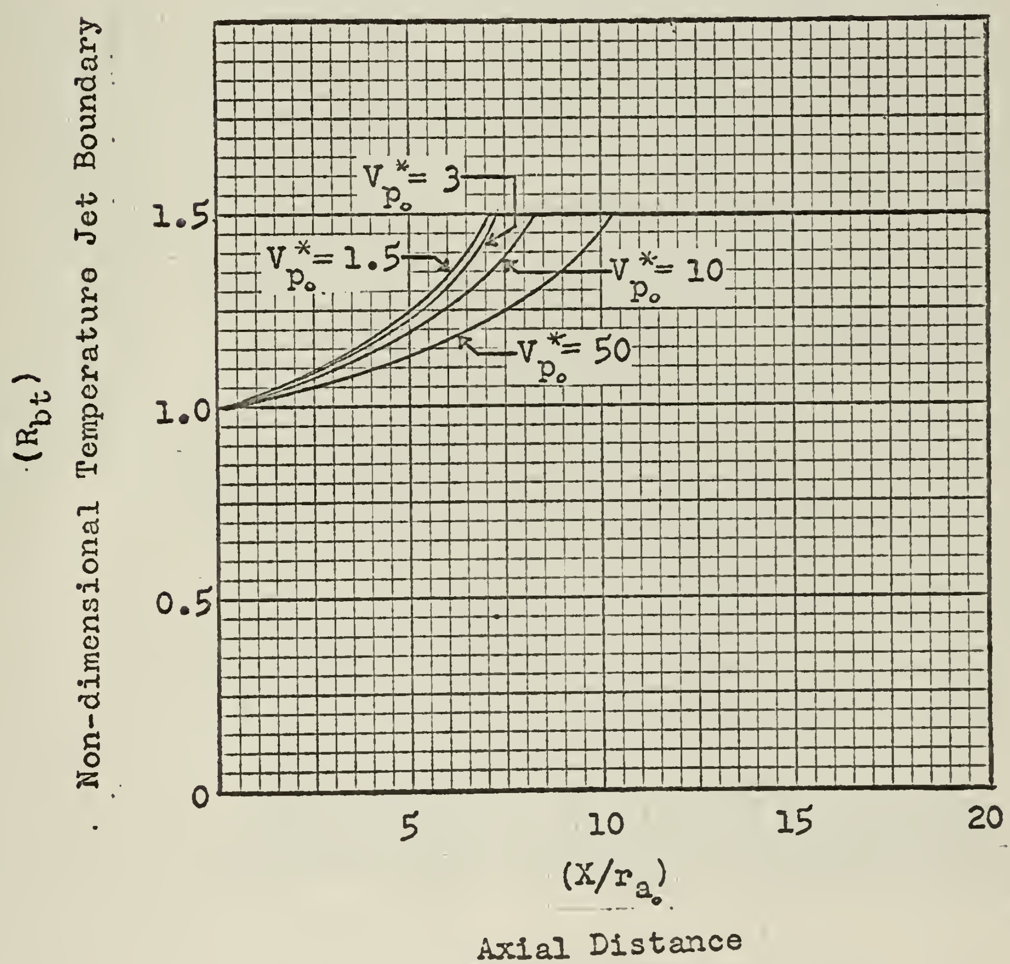
Axial Velocity Ratios vs. Axial Distance
for an Ejector Area Ratio of 2.25:1

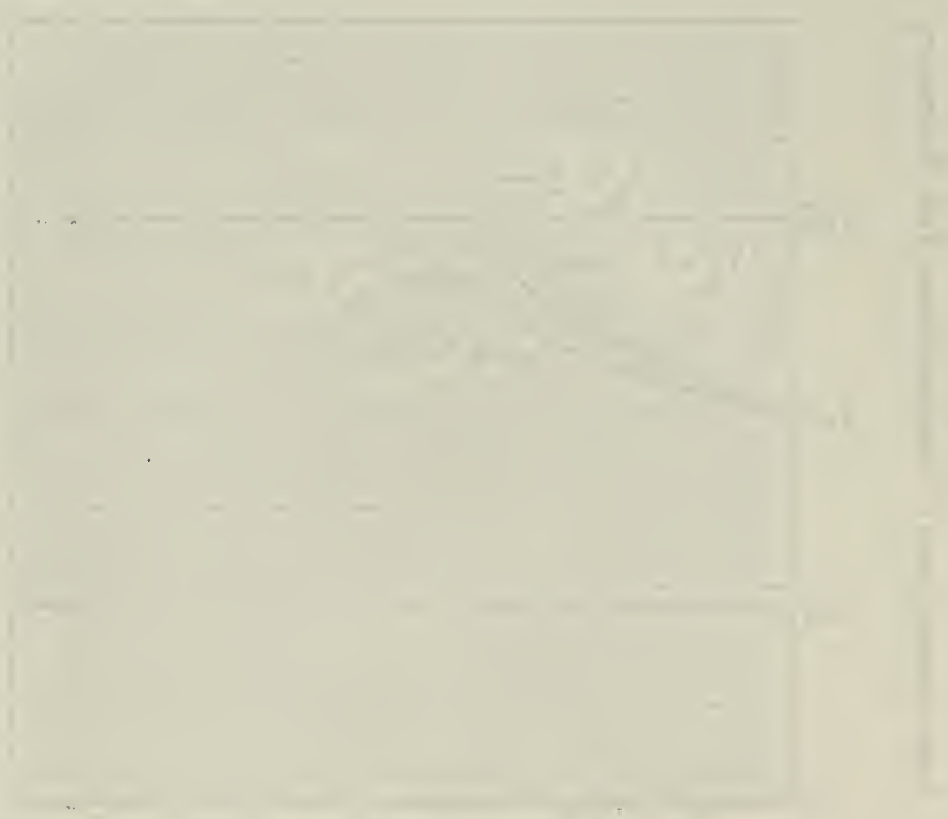


Peripheral Velocity Ratio vs. Axial Distance
for an Ejector Area Ratio of 2.25:1

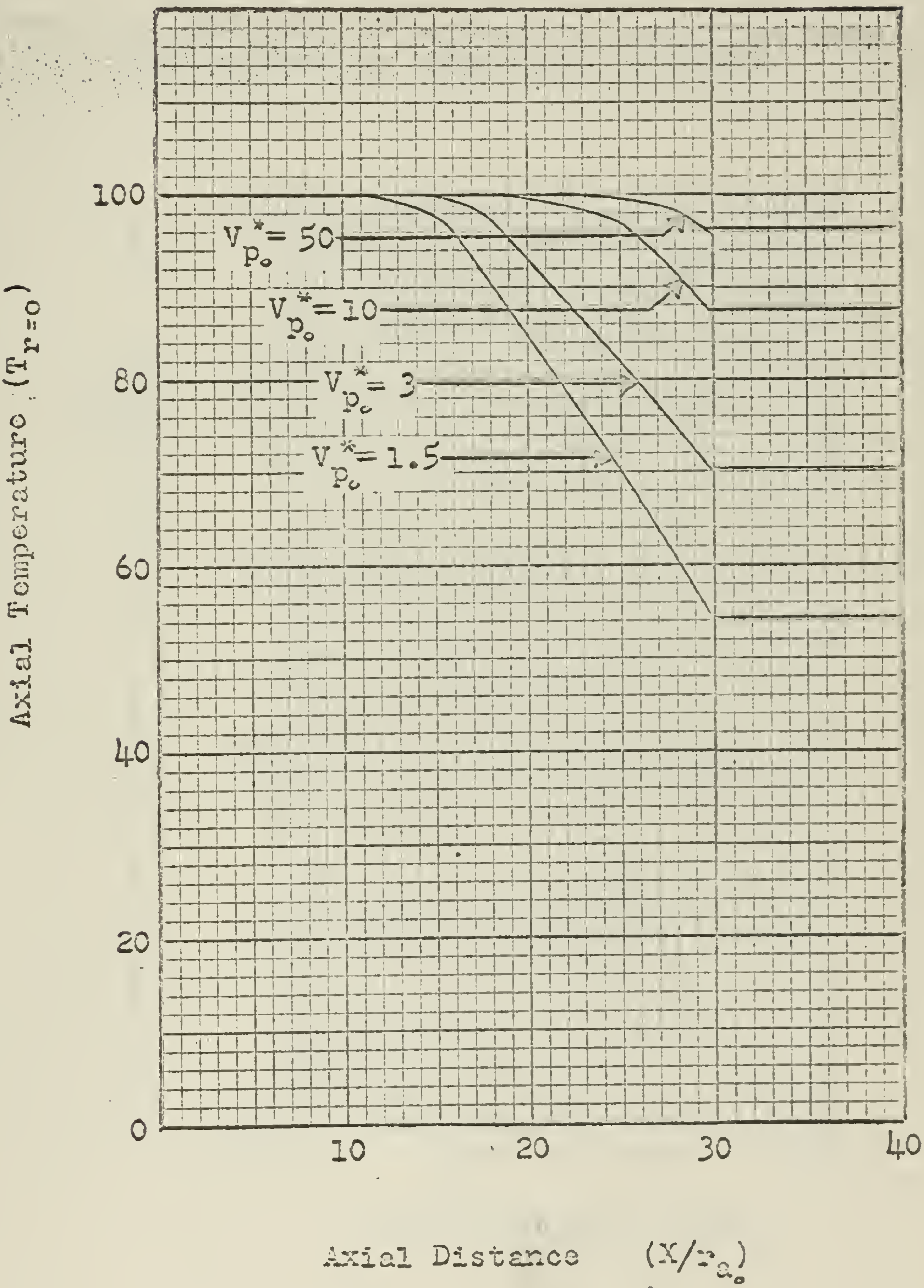


Non-dimensional Temperature Jet Boundary vs. Axial Distance for an Ejector Area Ratio of 2.25:1

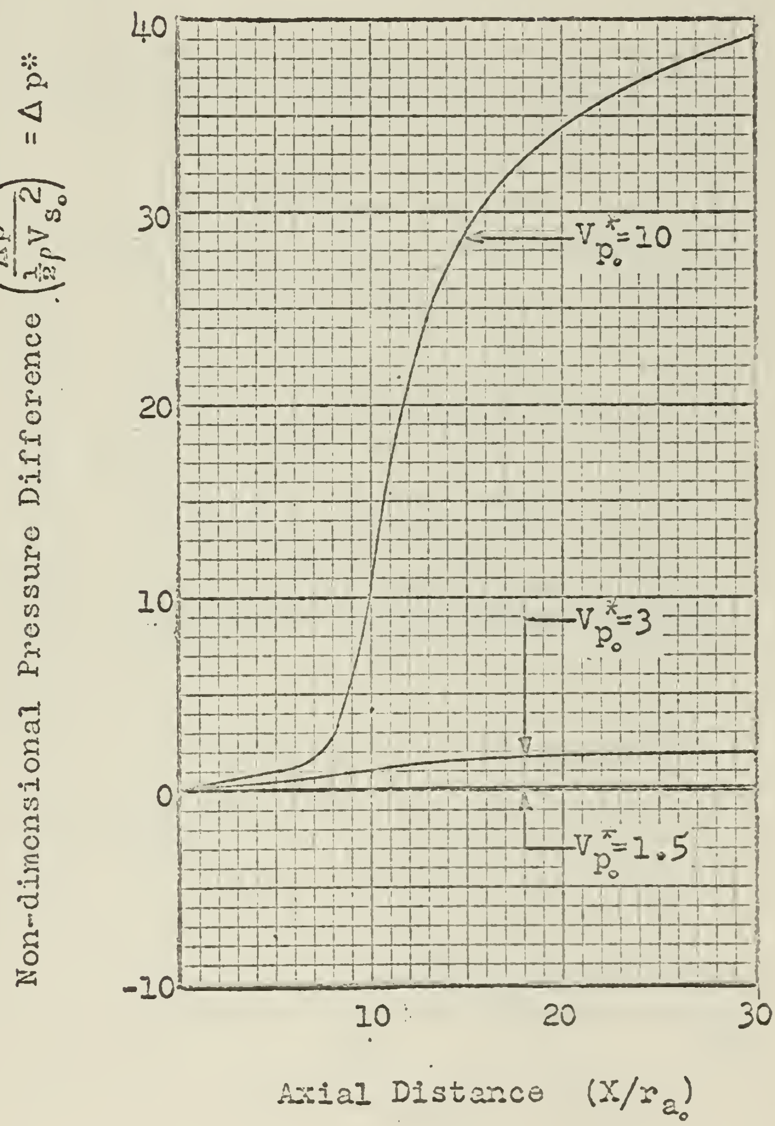




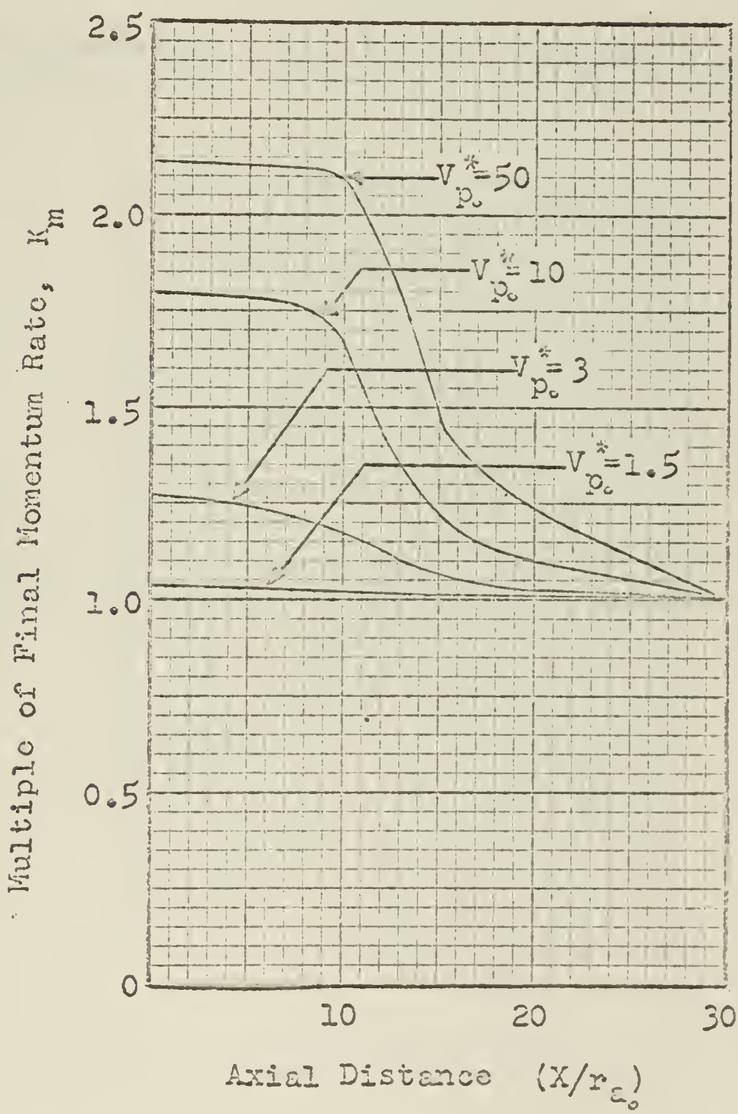
Axial Temperature vs. Axial Distance
for an Ejector Area Ratio of 2.25:1



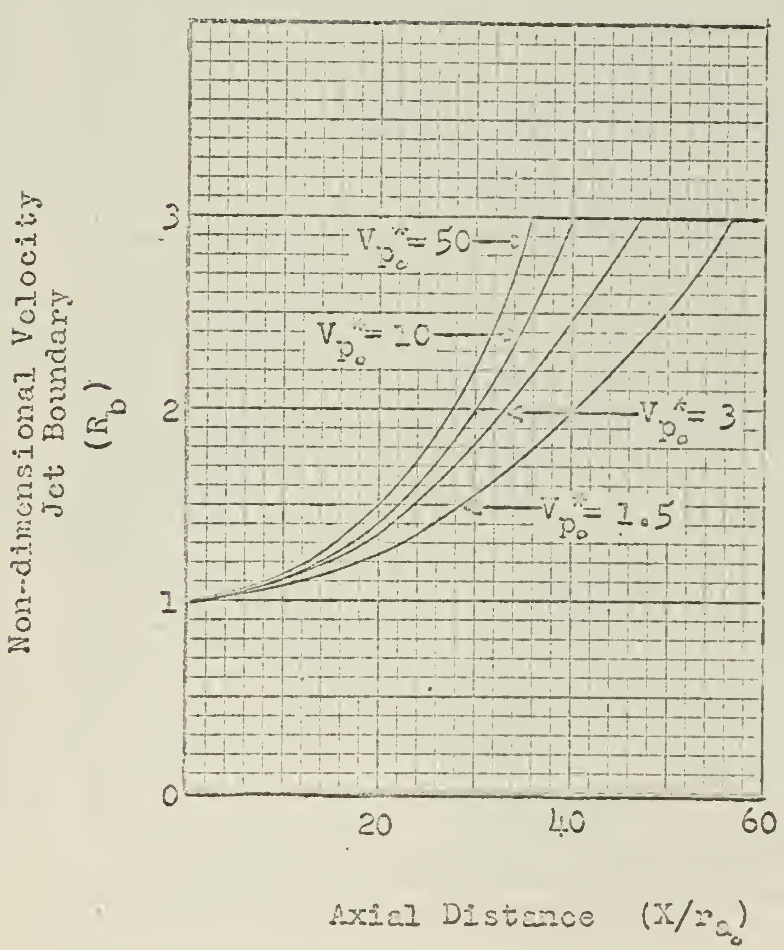
Non-dimensional Pressure Difference vs. Axial Distance
for an Ejector Area Ratio of 2.25:1



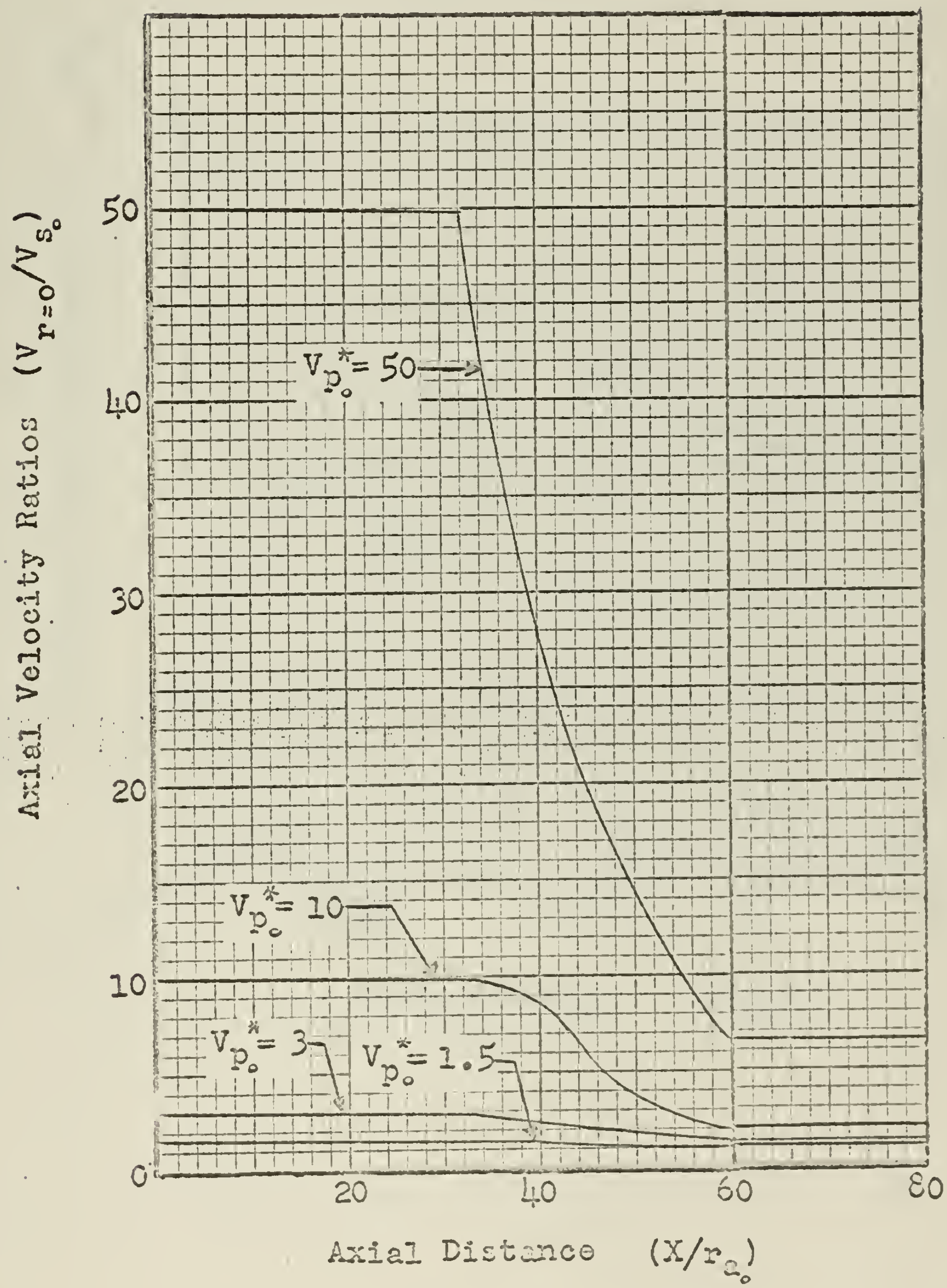
Multiple of Final Momentum Rate vs. Axial Distance
for an Ejector Area Ratio of 2.25:1



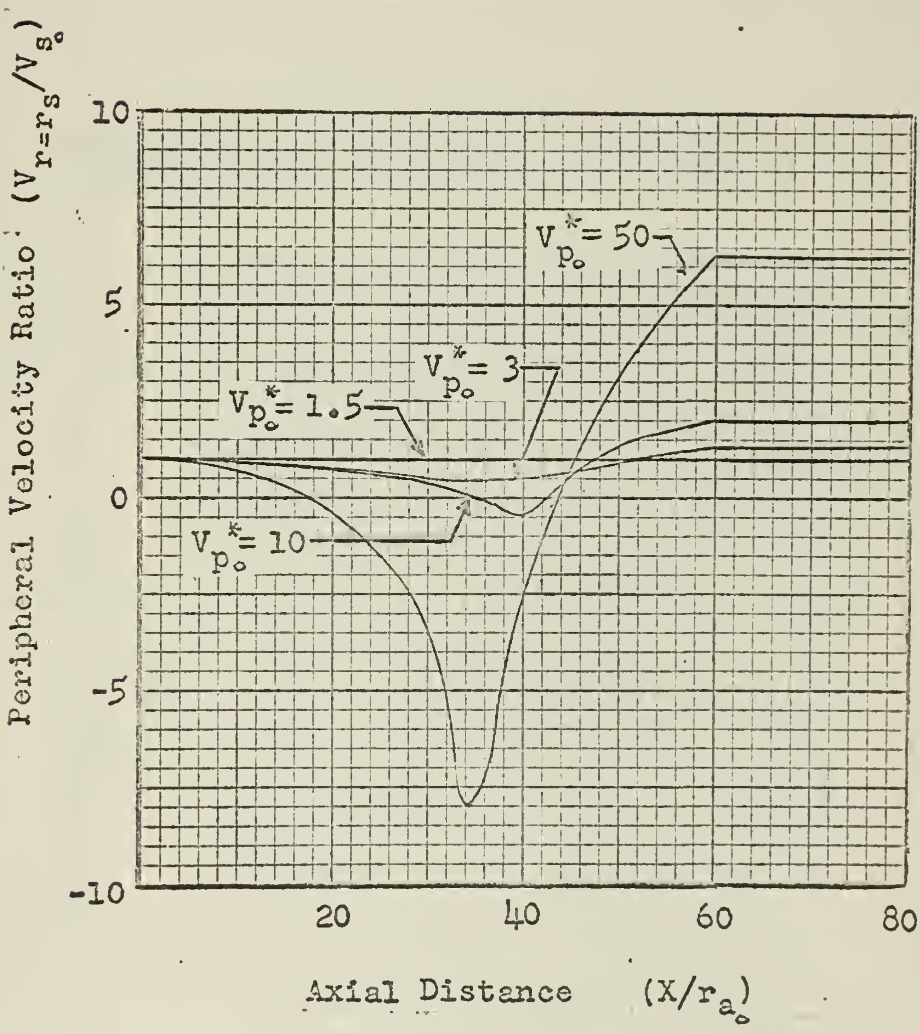
Non-dimensional Velocity Jet Boundary vs. Axial Distance for an Ejector Area Ratio of 9:1



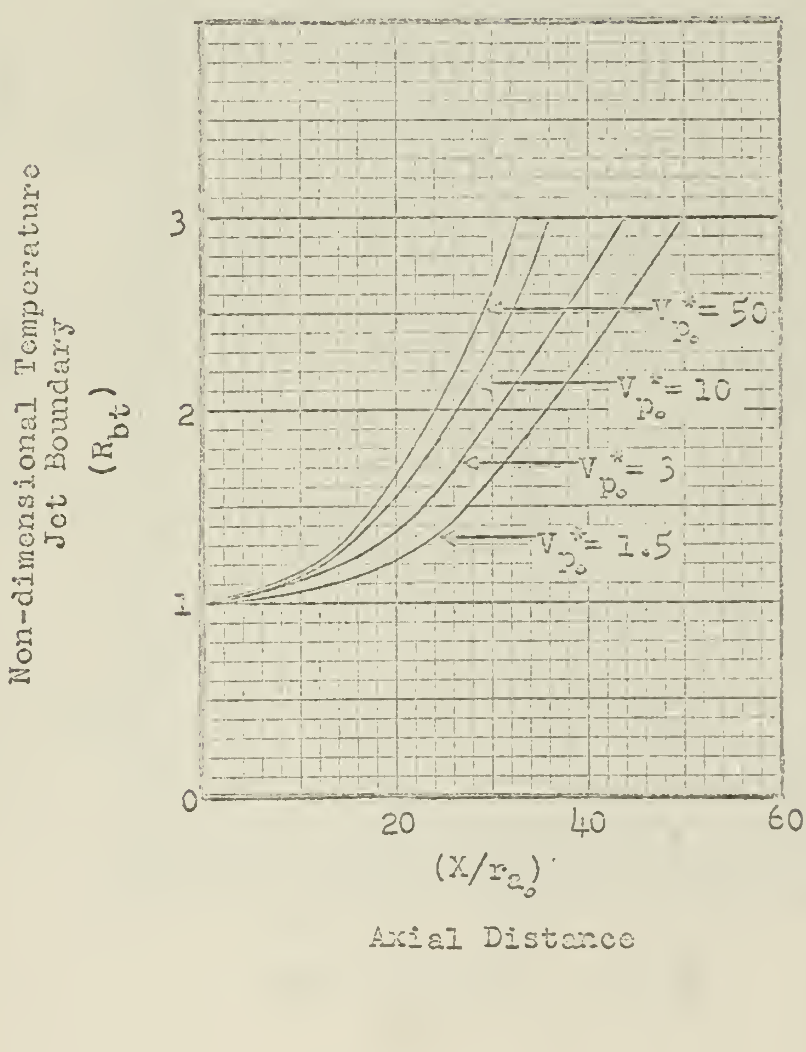
Axial Velocity Ratios vs. Axial Distance
for an Ejector Area Ratio of 9:1



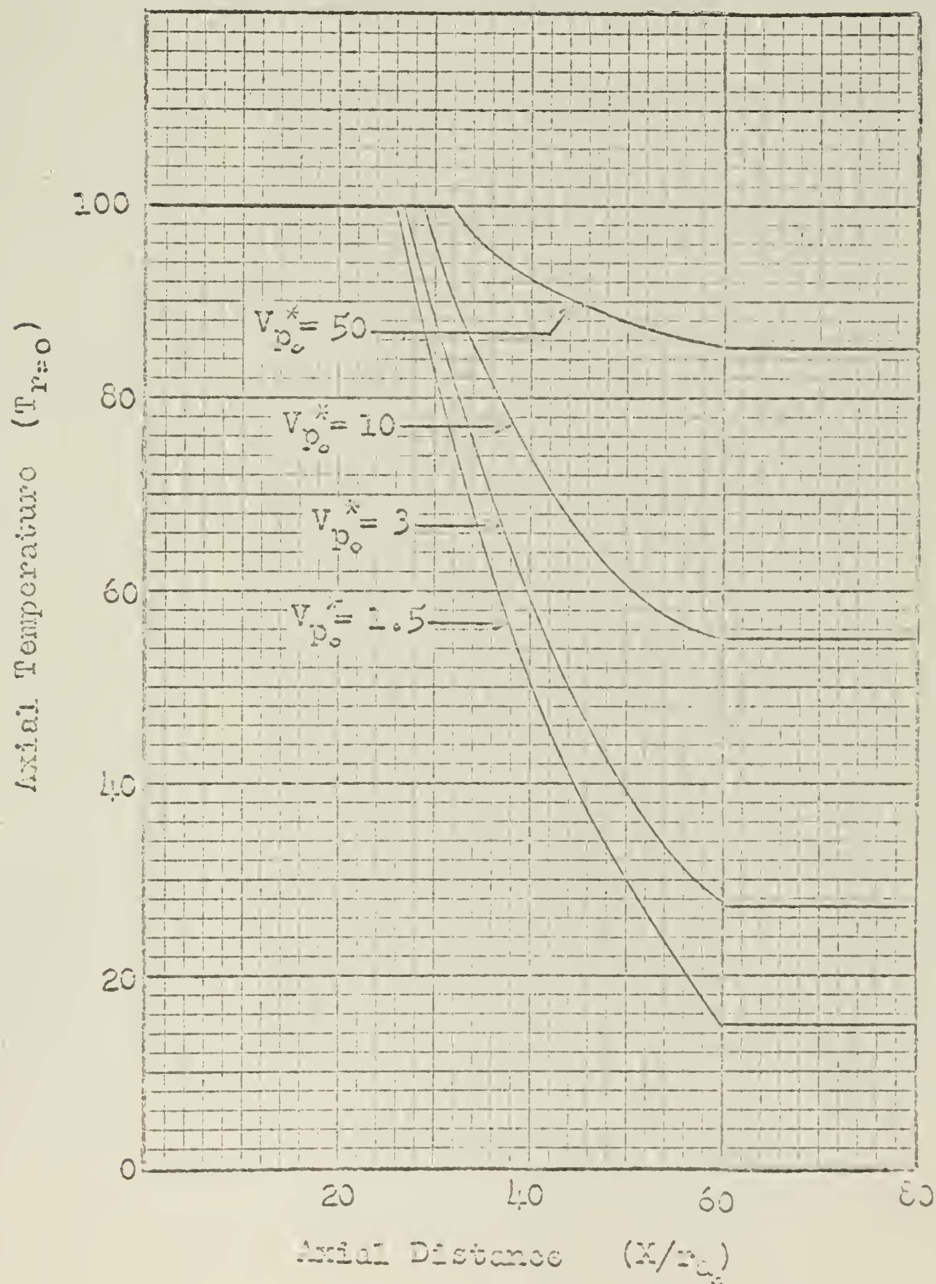
Peripheral Velocity Ratio vs. Axial Distance
for an Ejector Area Ratio of 9:1



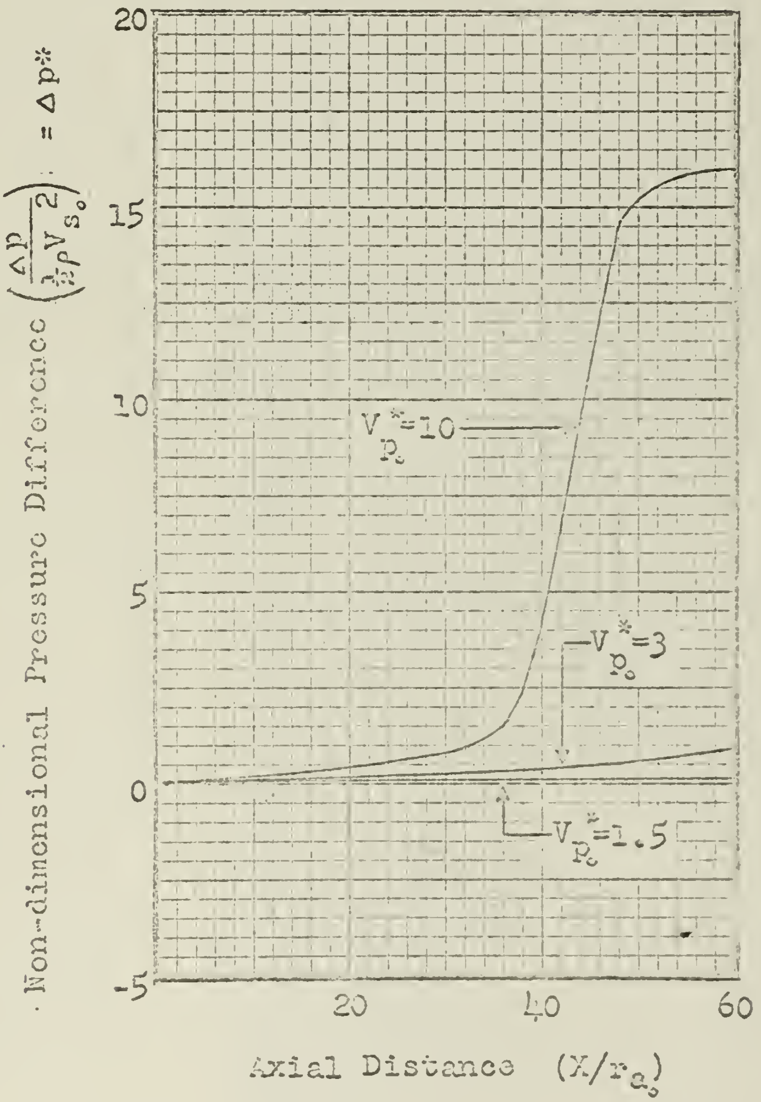
Non-dimensional Temperature Jet Boundary vs. Axial Distance for an Ejector Area Ratio of 9:1



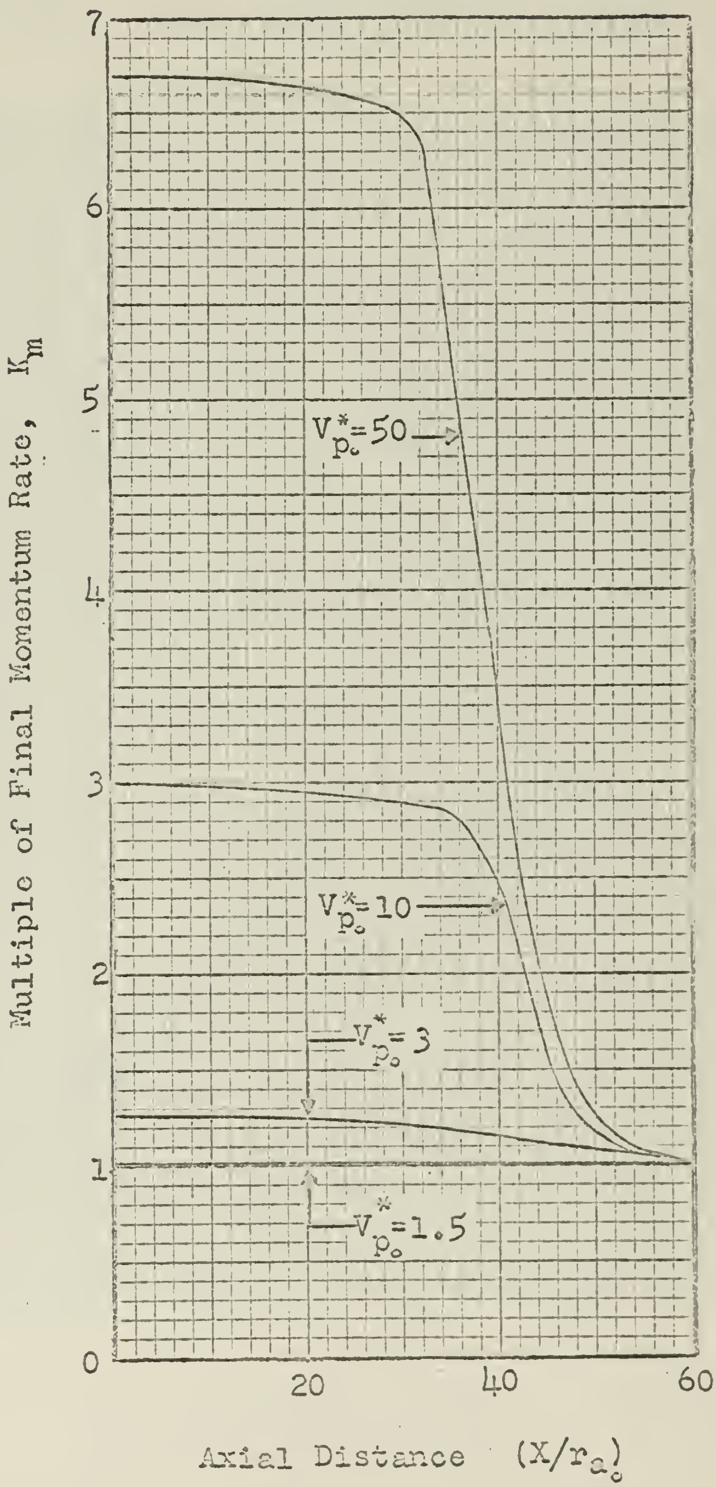
Axial Temperature vs. Axial Distance
for an Ejector Area Ratio of 9:1



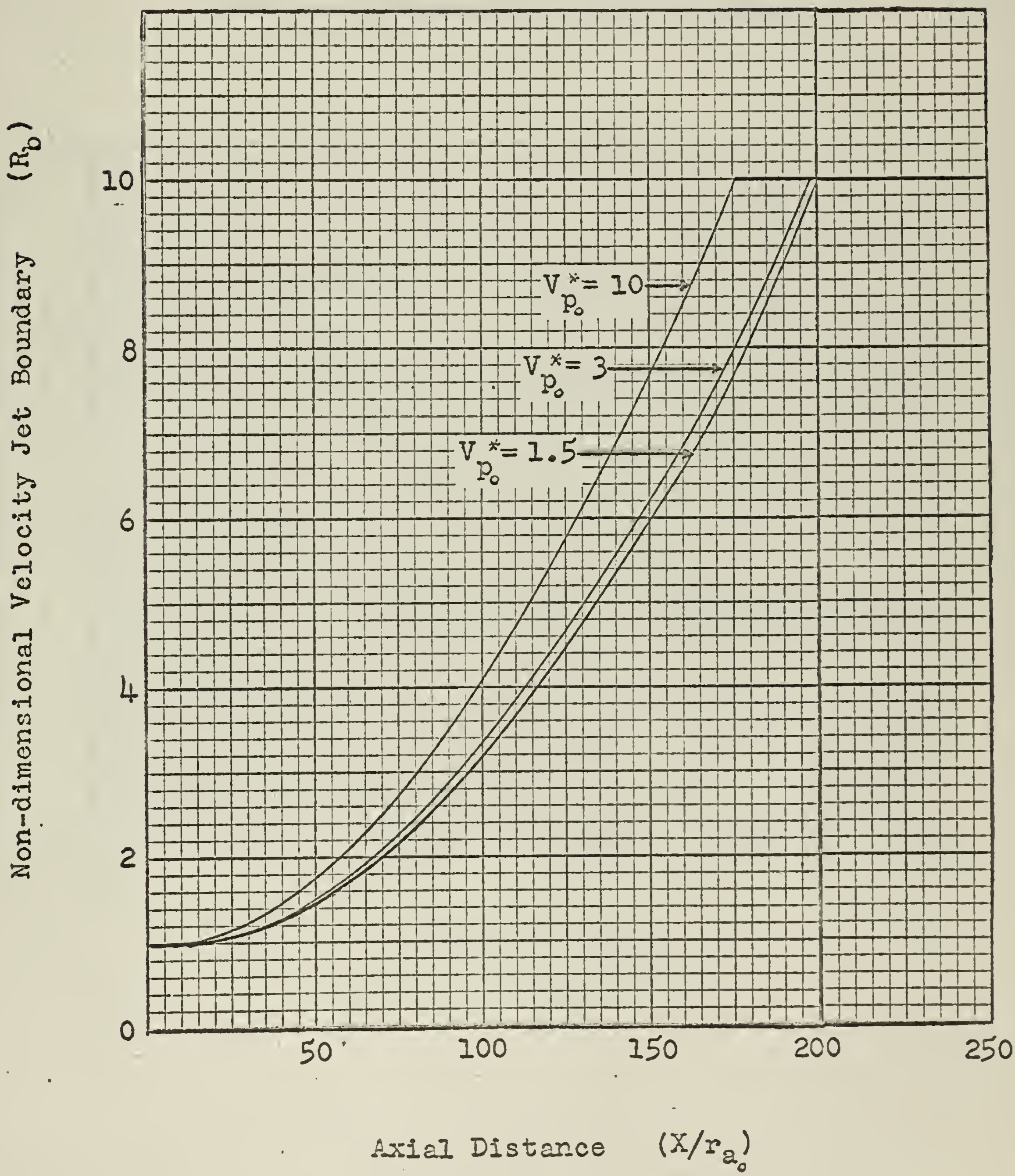
Non-dimensional Pressure Difference vs. Axial Distance for an Ejector Area Ratio of 9:1

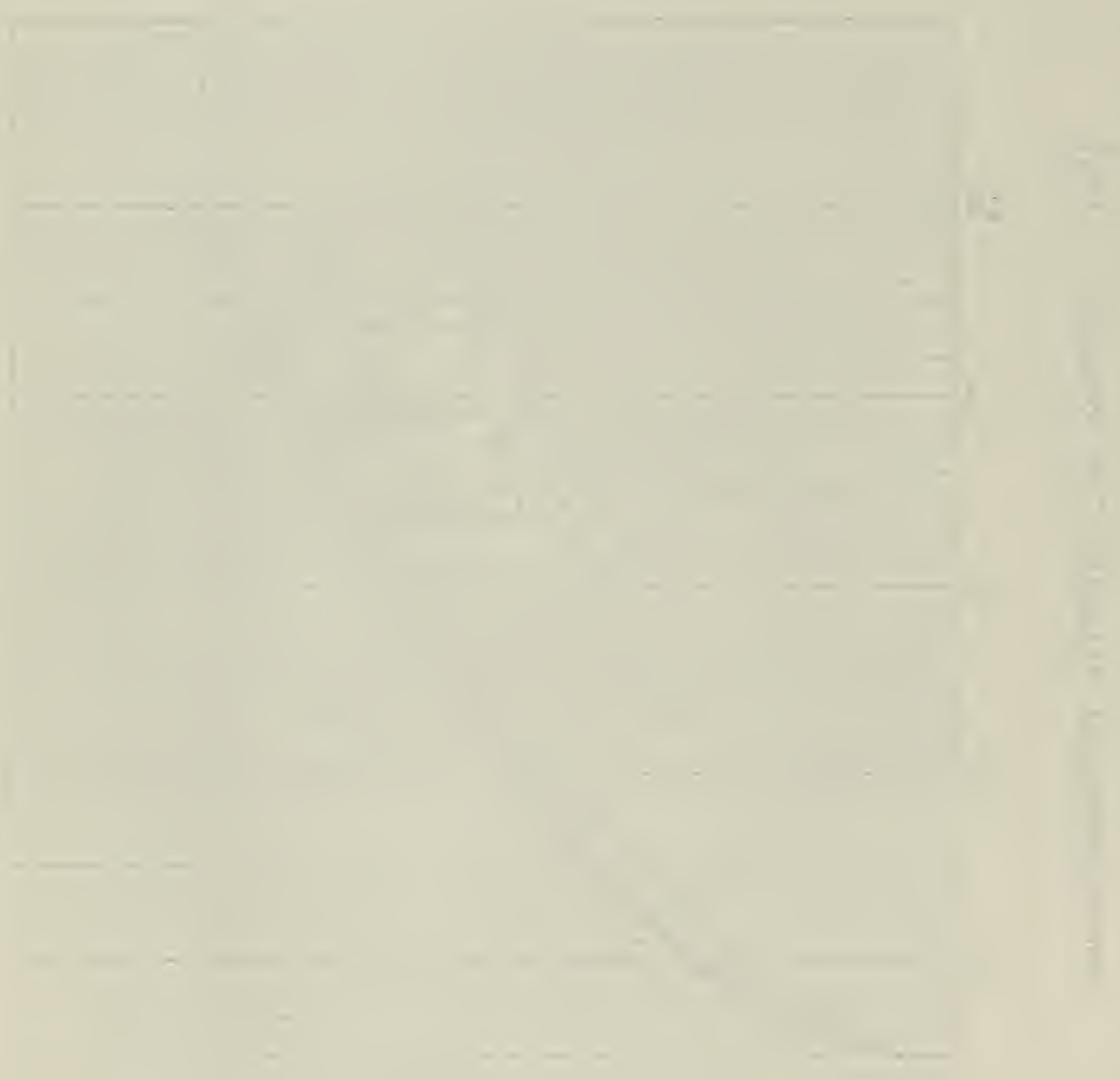


Multiple of Final Momentum Rate vs. Axial Distance
for an Ejector Area Ratio of 9:1

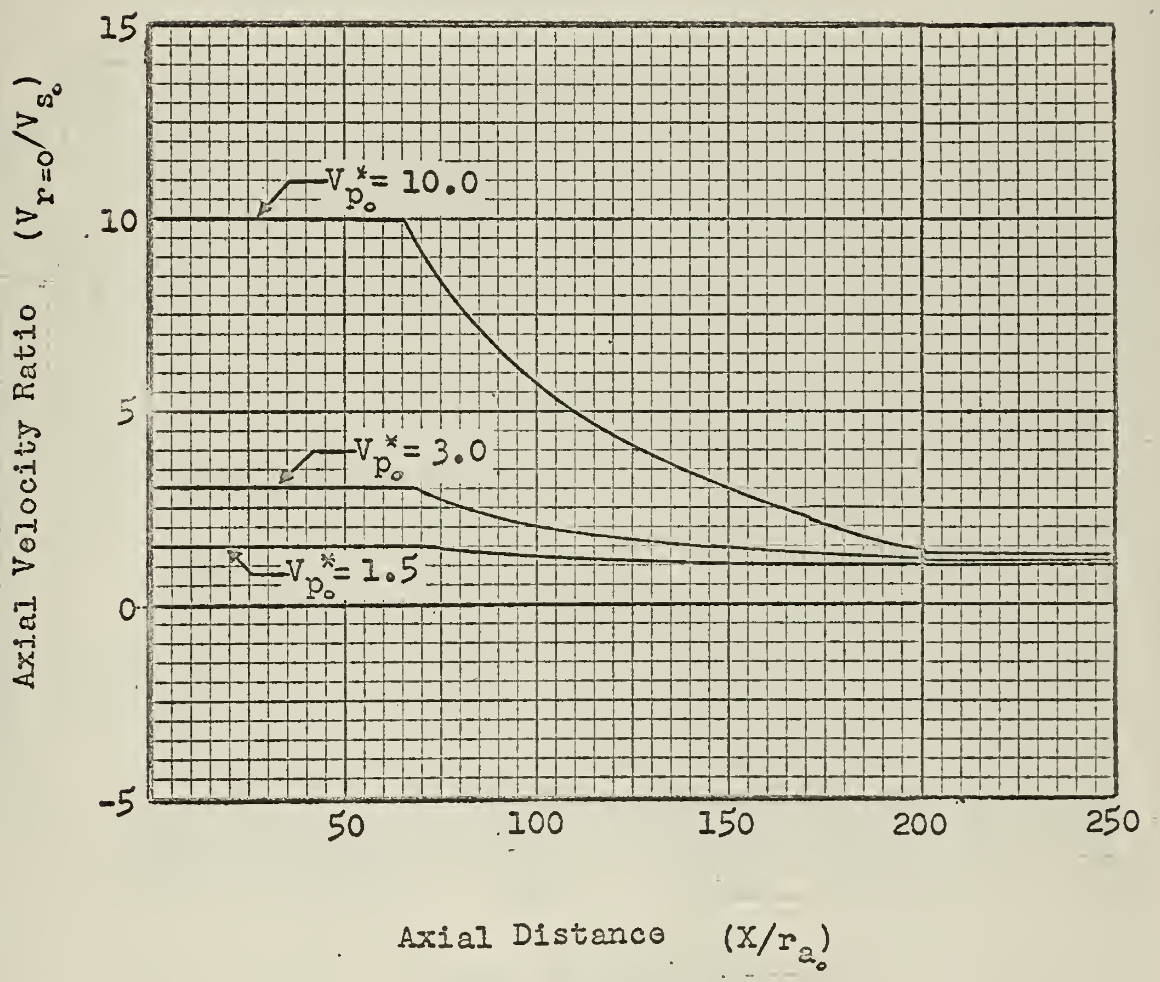


Non-dimensional Velocity Jet Boundary vs. Axial Distance for an Ejector Area Ratio of 100:1

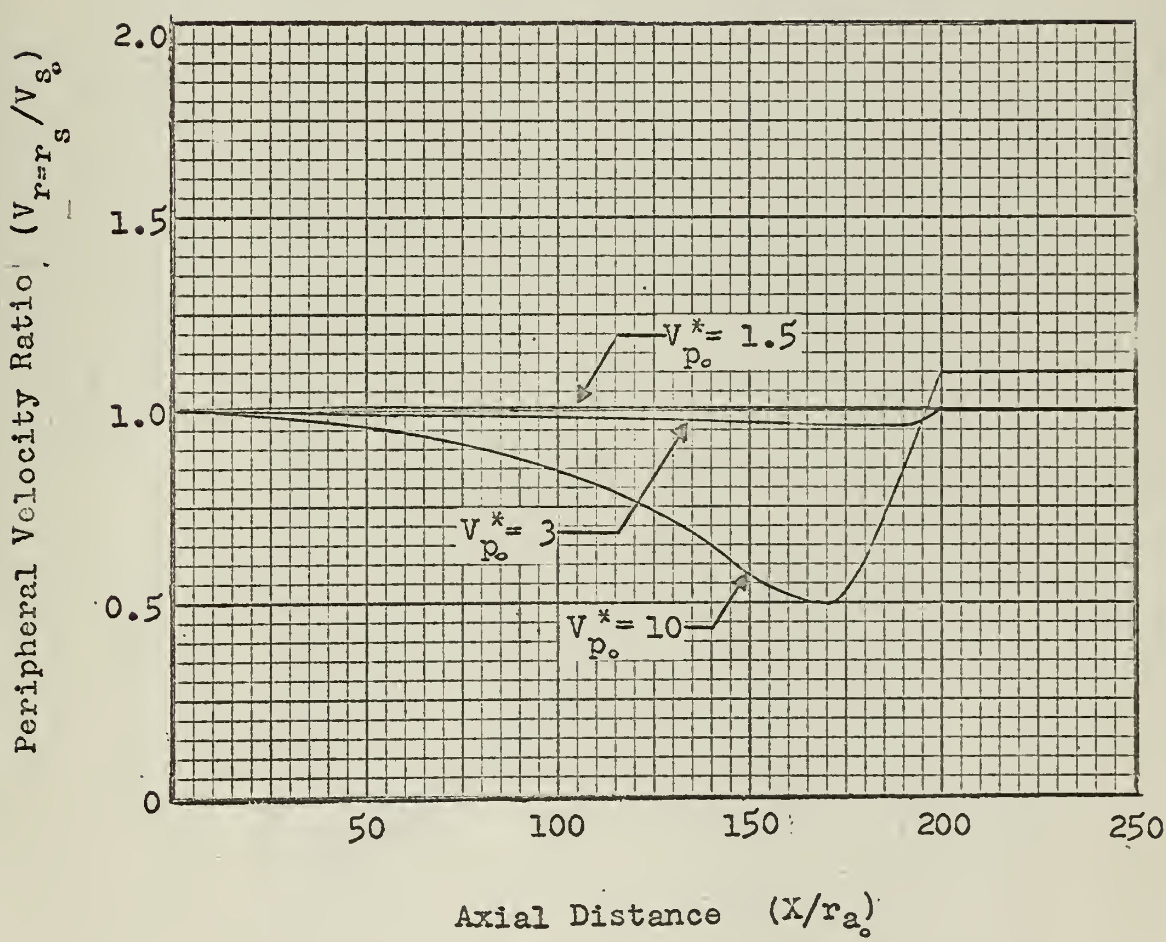




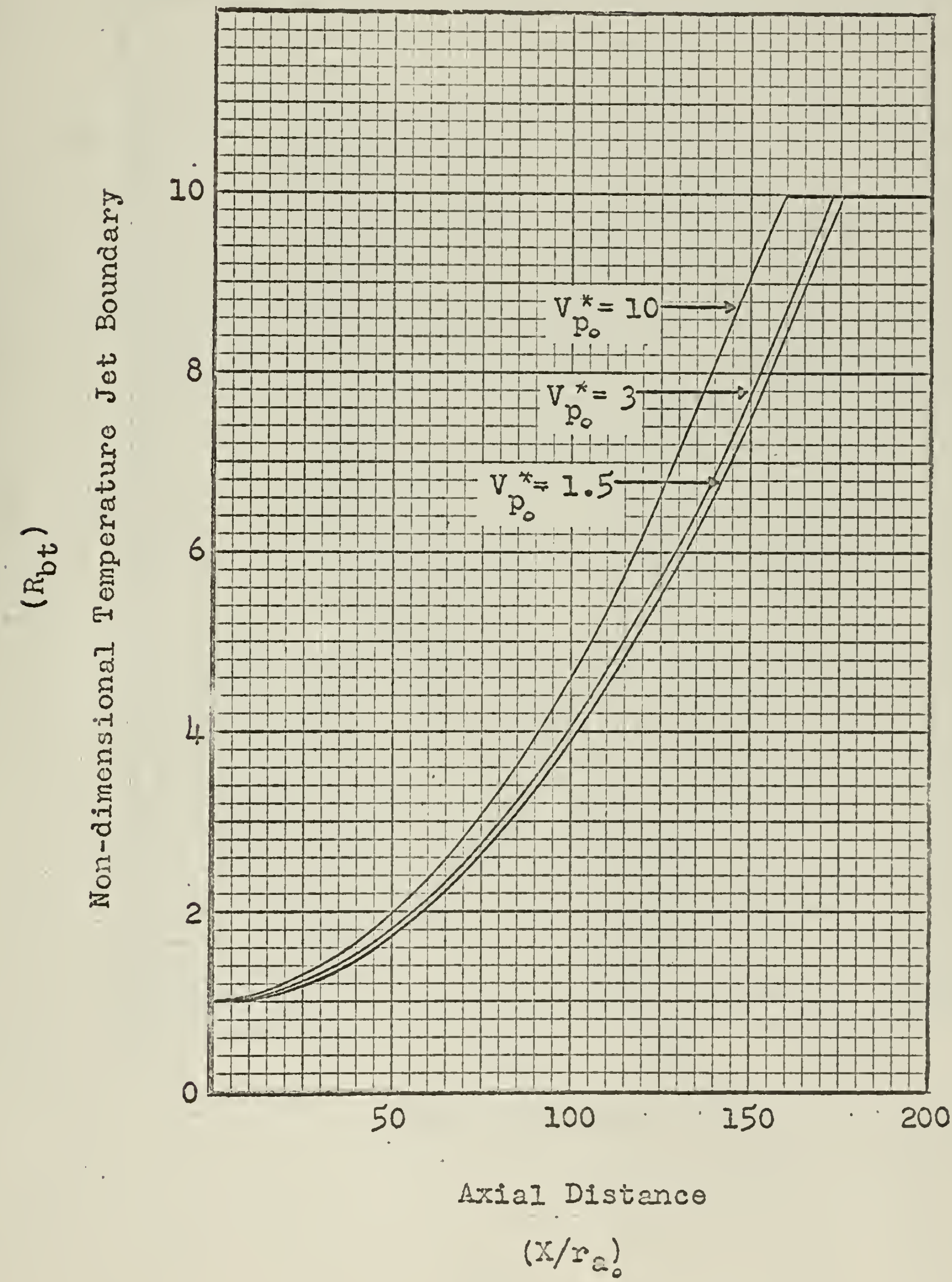
Axial Velocity Ratio vs. Axial Distance
for an Ejector Area Ratio of 100:1



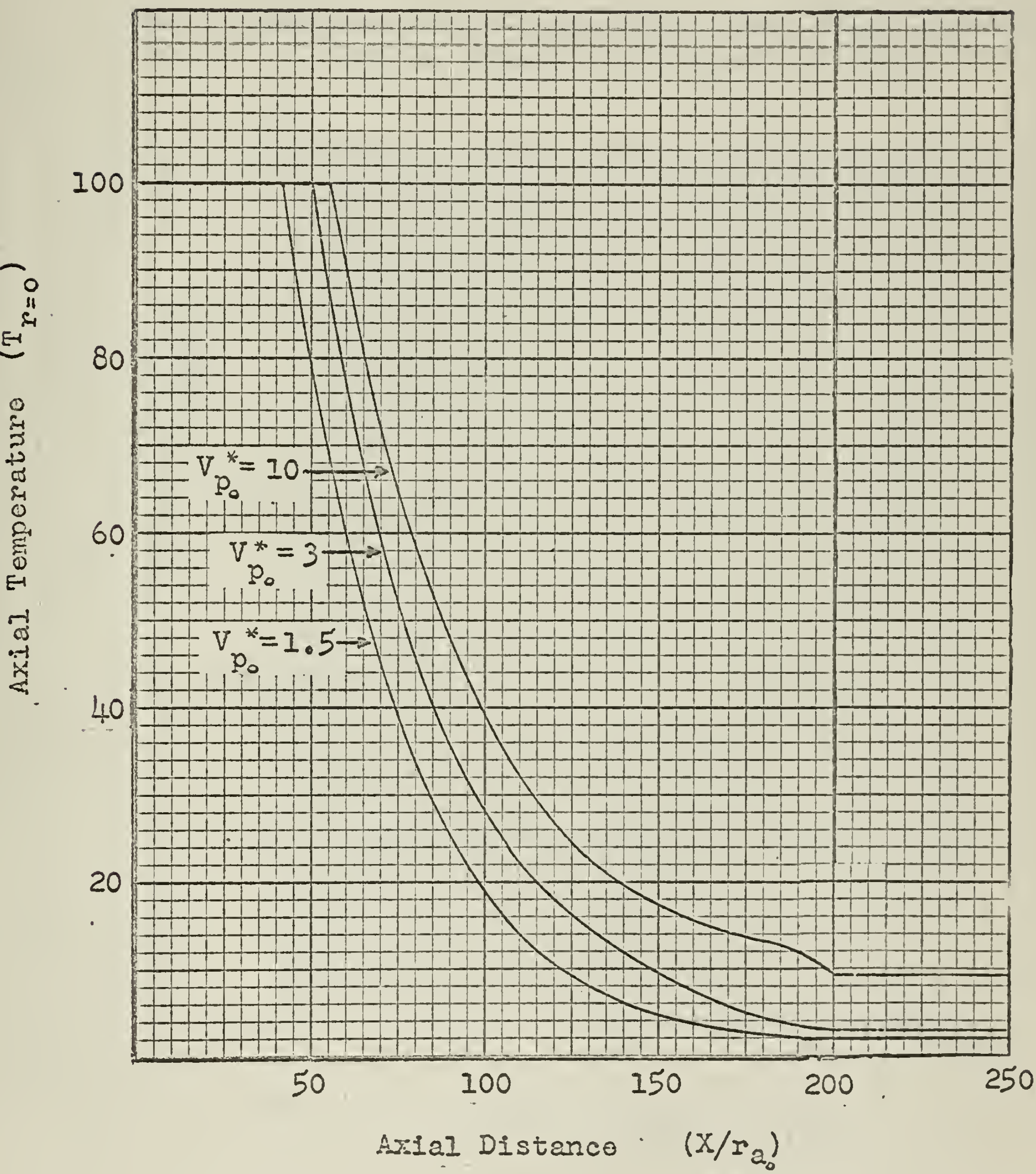
Peripheral Velocity Ratio vs. Axial Distance
for an Ejector Area Ratio of 100:1



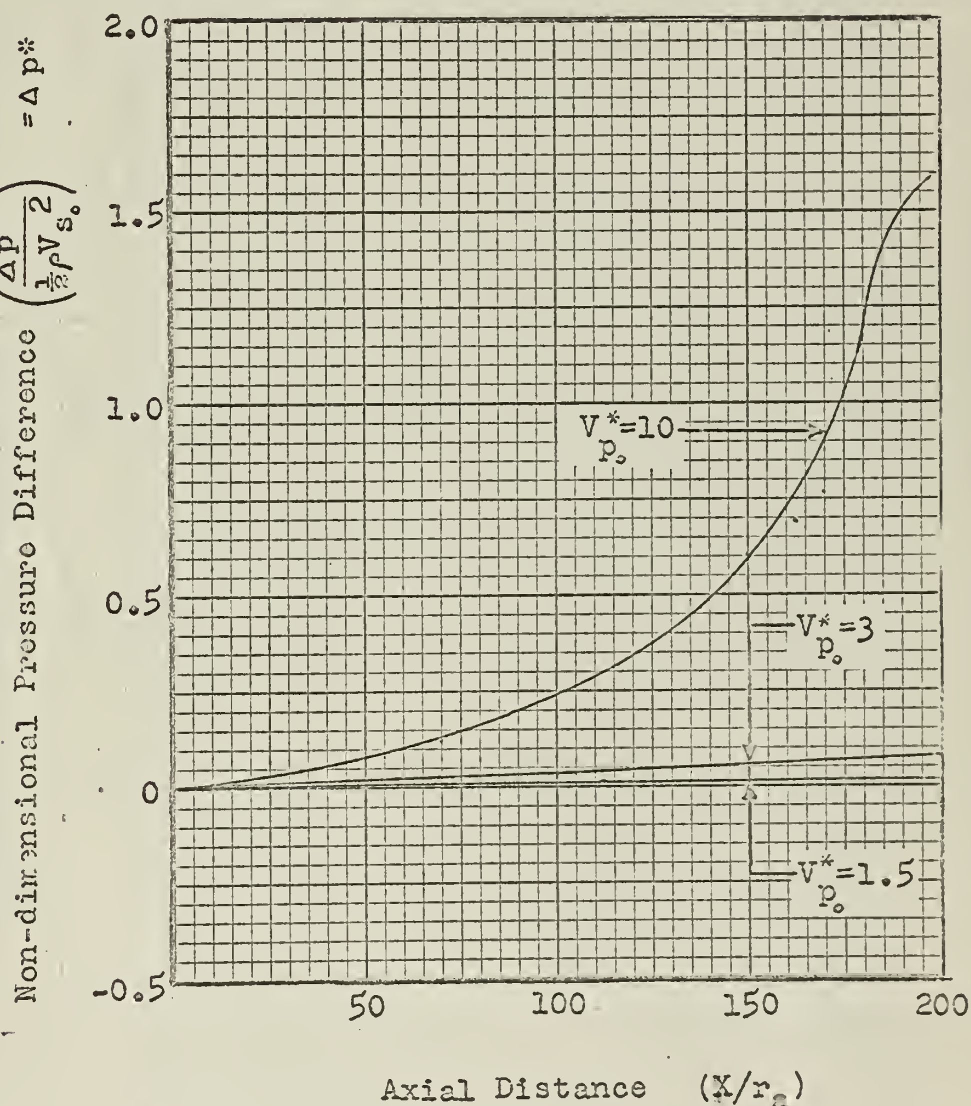
Non-dimensional Temperature Jet Boundary vs. Axial Distance for an Ejector Area Ratio of 100:1



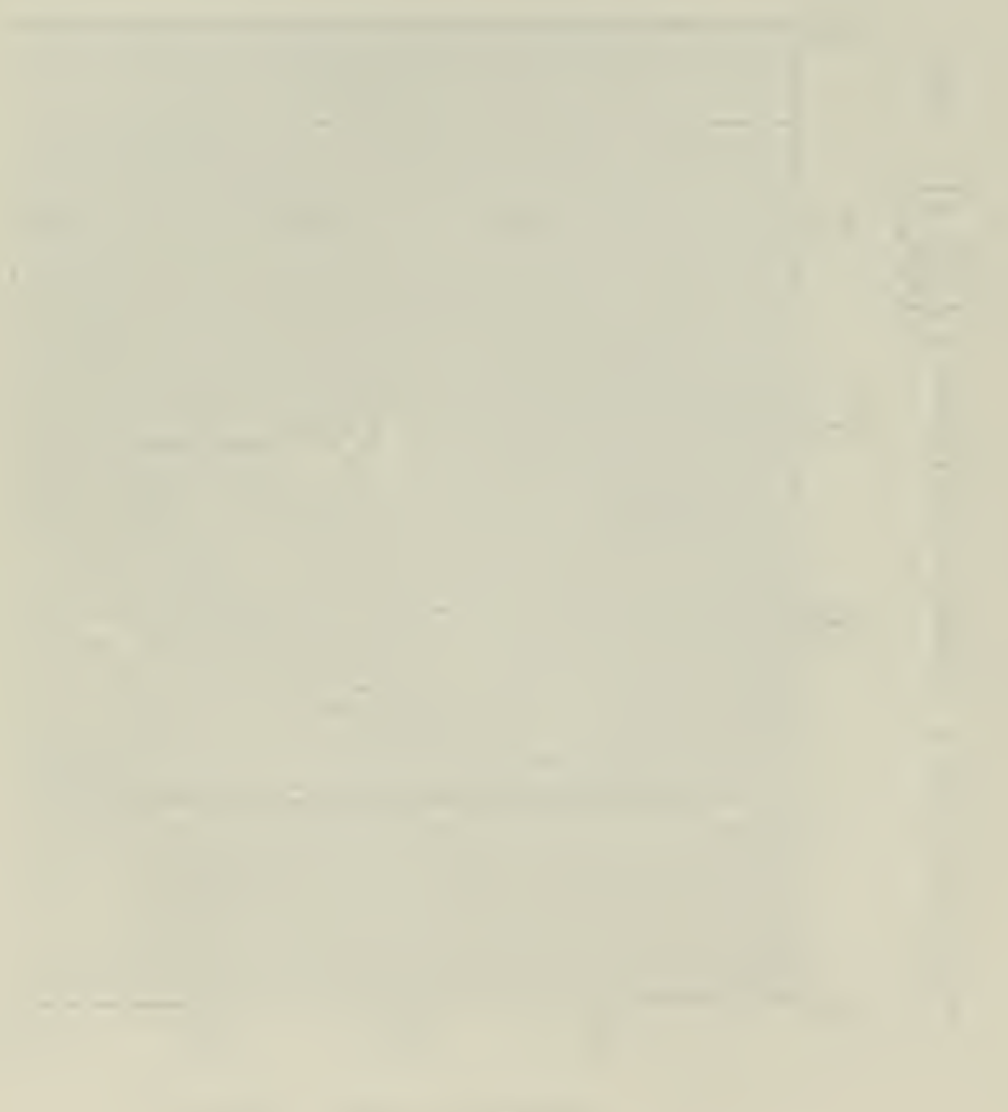
Axial Temperature vs. Axial Distance
for an Ejector Area Ratio of 100:1.



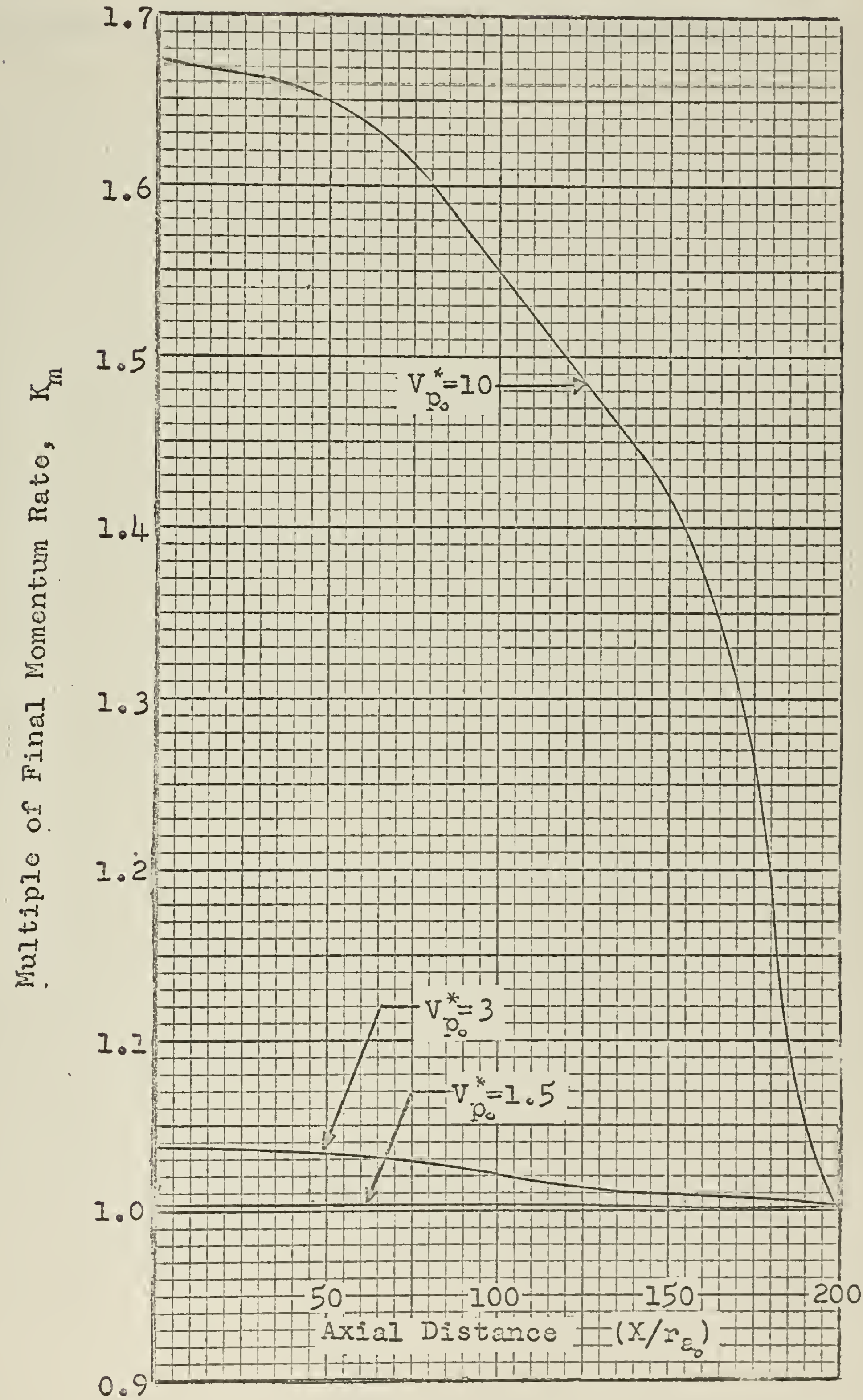
Non-dimensional Pressure Difference vs. Axial Distance for an Ejector Area Ratio of 100:1

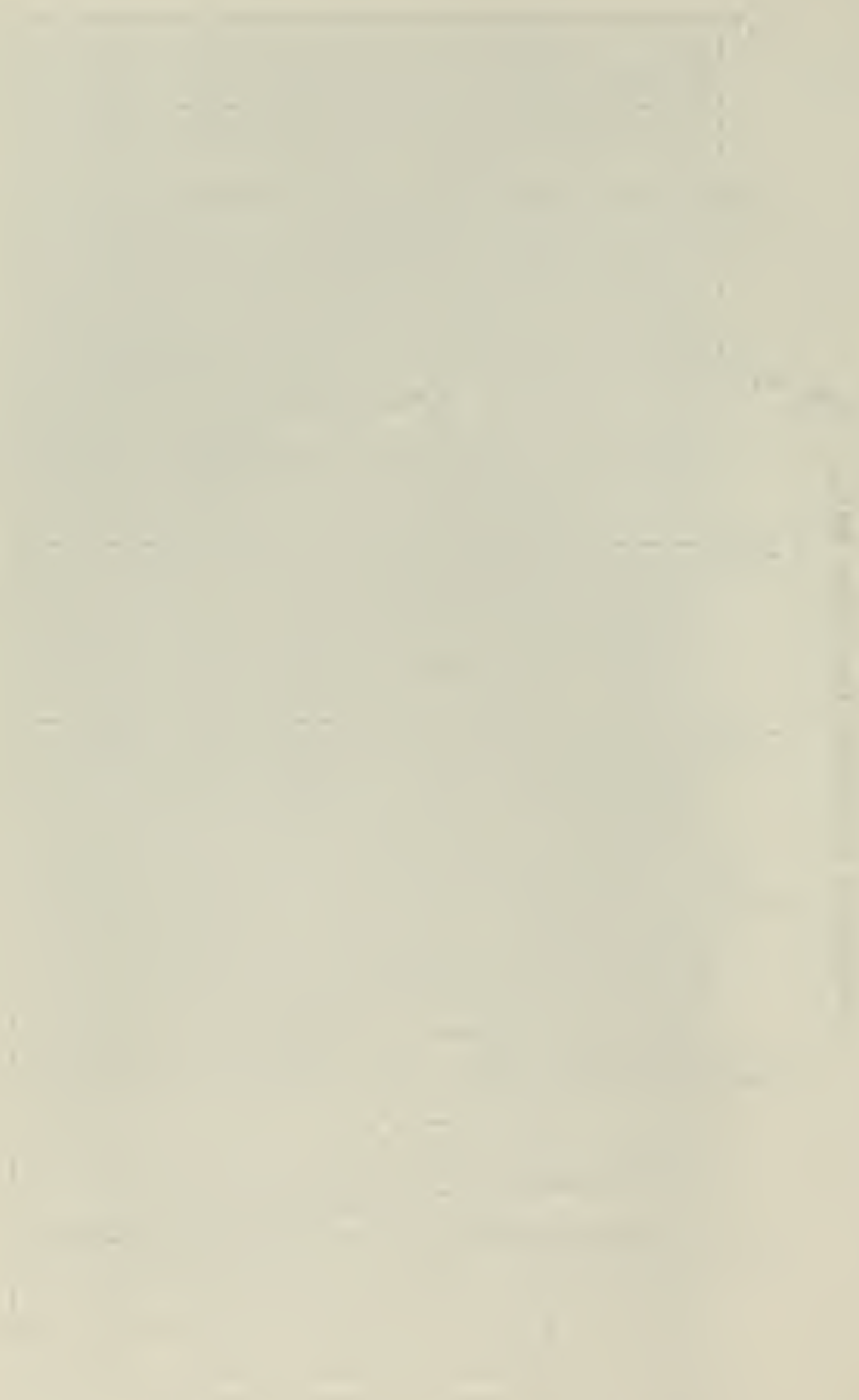


[illegible]



Multiple of Final Momentum Rate vs. Axial Distance
for an Ejector Area Ratio of 100:1





APPENDIX III

Appendix III contains edited data from which the graphs of Appendix II were drawn. The headings are the same as defined in the section of Symbols and Abbreviations.

Edited Data for Initial Conditions of
Central Velocity Ratio of 1.5 and Peripheral Radius Ratio of 1.5

X/r_a	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	1.5000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	1.0413
3.9	.9000	1.1075	1.4987	.9980	.8596	1.1398	100.0000	.0000	.0089	1.0384
5.6	.8000	1.2222	1.4950	.9925	.6866	1.2888	100.0000	.0000	.0199	1.0347
6.9	.7000	1.3363	1.4905	.9857	.4243	1.4378	100.0000	.0000	.0311	1.0309
8.0	.6000	1.4514	1.4852	.9776	.2622	1.5000	100.0000	3.1283	.0427	1.0270
9.1	.5000	1.5000	1.4837	.9938	.1621	1.5000	100.0000	3.3169	.0593	1.0215
10.2	.4000	1.5000	1.4789	1.0306	.1002	1.5000	100.0000	4.6390	.0738	1.0166
11.3	.3000	1.5000	1.4750	1.0597	.0619	1.5000	100.0000	5.8913	.0849	1.0129
15.0	.2000	1.5000	1.4721	1.0830	.0000	1.5000	99.4874	7.1436	.0933	1.0101
18.5	.1000	1.5000	1.4700	1.1019	.0000	1.5000	99.1098	7.2033	.0998	1.0079
22.5	.0000	1.5000	1.3700	1.1591	.0000	1.5000	81.1250	25.4019	.1181	1.0018
26.1	.0000	1.5000	1.2700	1.2015	.0000	1.5000	63.1402	45.0112	.1245	.9996
30.0	.0000	1.5000	1.1700	1.2437	.0000	1.5000	45.1555	63.5511	.1247	.9996

Edited Data for Initial Conditions of
Central Velocity Ratio of 3.0 and Peripheral Radius Ratio of 1.5

X/r_{a_0}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	3.0000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	1.2768
4.6	.9000	1.1463	2.9793	.9361	.8470	1.1902	100.0000	.0000	.1278	1.2589
6.5	.8000	1.2953	2.9545	.8537	.6785	1.3839	100.0000	.0000	.2747	1.2383
8.1	.7000	1.4505	2.9242	.7423	.5435	1.5000	100.0000	10.9593	.4516	1.2135
9.7	.6000	1.5000	2.8625	.8866	.4354	1.5000	100.0000	17.3726	.8466	1.1582
10.8	.5000	1.5000	2.8056	1.0898	.3487	1.5000	100.0000	26.2395	1.1676	1.1132
12.8	.4000	1.5000	2.7685	1.2338	.2794	1.5000	100.0000	30.3976	1.3732	1.0844
14.3	.3000	1.5000	2.7426	1.3417	.2238	1.5000	100.0000	32.7374	1.5154	1.0645
15.8	.2000	1.5000	2.7236	1.4255	.0000	1.5000	99.3454	37.4182	1.6187	1.0500
17.4	.1000	1.5000	2.7092	1.4921	.0000	1.5000	98.8493	38.8691	1.6963	1.0391
19.0	.0000	1.5000	2.6092	1.5835	.0000	1.5000	95.4042	41.4353	1.8017	1.0243
20.5	.0000	1.5000	2.5092	1.6258	.0000	1.5000	91.9590	45.3568	1.8474	1.0179
22.0	.0000	1.5000	2.4092	1.6681	.0000	1.5000	88.5139	49.6494	1.8863	1.0125
23.5	.0000	1.5000	2.3092	1.7104	.0000	1.5000	85.0688	54.0148	1.9184	1.0080
24.9	.0000	1.5000	2.2092	1.7528	.0000	1.5000	81.6236	58.1286	1.9432	1.0045
26.2	.0000	1.5000	2.1092	1.7951	.0000	1.5000	78.1785	61.5346	1.9617	1.0019
27.5	.0000	1.5000	2.0092	1.8374	.0000	1.5000	74.7334	65.1401	1.9735	1.0003
28.7	.0000	1.5000	1.9092	1.8797	.0000	1.5000	71.2882	68.9570	1.9784	.9996
30.0	.0000	1.5000	1.8092	1.9220	.0000	1.5000	67.8431	72.5203	1.9765	.9998

Edited Data for Initial Conditions of
Central Velocity Ratio of 10.0 and Peripheral Radius Ratio of 1.5

X/r_{a_o}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	10.0000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	1.8000
5.3	.9000	1.1560	9.9693	.6213	.8566	1.2028	100.0000	.0000	1.0681	1.7786
7.7	.8000	1.3350	9.9487	-.1512	.7106	1.4355	100.0000	.0000	1.4735	1.7705
10.2	.7000	1.5000	9.4931	-.6303	.5895	1.5000	100.0000	42.3383	10.3283	1.5934
12.7	.6000	1.5000	8.7561	1.0354	.4890	1.5000	100.0000	60.1028	23.7104	1.3258
15.1	.5000	1.5000	8.4268	1.9226	.4056	1.5000	100.0000	70.7816	29.3393	1.2132
17.6	.4000	1.5000	8.2319	2.5090	.3365	1.5000	100.0000	73.1986	32.5725	1.1485
20.1	.3000	1.5000	8.1018	2.9326	.0000	1.5000	99.5527	76.1276	34.6869	1.1063
22.6	.2000	1.5000	8.0095	3.2540	.0000	1.5000	99.2354	77.5881	36.1660	1.0767
25.1	.1000	1.5000	7.9415	3.5053	.0000	1.5000	99.0016	77.5988	37.2453	1.0551
27.6	.0000	1.5000	5.9415	4.6009	.0000	1.5000	92.1258	82.5090	39.7071	1.0059
30.0	.0000	1.5000	4.6944	4.6944	.0000	1.5000	87.8288	87.8288	39.6281	1.0000

Edited Data for Initial Conditions of
Central Velocity Ratio of 50.0 and Peripheral Radius Ratio of 1.5

X/r_{a_0}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	50.0000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	2.1427
6.7	.9000	1.1610	49.9689	-1.4517	.8618	1.2093	100.0000	.0000	15.0757	2.1281
9.8	.8000	1.3400	49.6806	-5.5528	.7583	1.4420	100.0000	.0000	43.0149	2.1012
12.8	.7000	1.5000	46.7428	-7.1640	.6673	1.5000	100.0000	72.2358	336.7149	1.8182
15.8	.6000	1.5000	41.9593	2.5901	.5871	1.5000	100.0000	85.4229	756.8047	1.4133
19.0	.5000	1.5000	40.0679	7.2986	.5166	1.5000	100.0000	94.1778	910.3820	1.2653
22.0	.4000	1.5000	38.9865	10.3271	.4546	1.5000	100.0000	94.9378	995.0087	1.1838
22.8	.3000	1.5000	38.2799	12.4847	.0000	1.5000	99.8937	95.1514	1049.0897	1.1316
23.5	.2000	1.5000	37.7825	14.1063	.0000	1.5000	99.8188	94.9089	1086.6187	1.0955
24.5	.1000	1.5000	37.4198	15.3703	.0000	1.5000	99.7642	94.6664	1113.6232	1.0694
25.0	.0000	1.5000	35.4198	17.4271	.0000	1.5000	99.4633	93.0335	1131.3694	1.0523
25.8	.0000	1.5000	33.4198	18.2735	.0000	1.5000	99.1623	92.0432	1147.2041	1.0371
26.5	.0000	1.5000	31.4198	19.1199	.0000	1.5000	98.8614	92.6695	1160.3187	1.0244
27.3	.0000	1.5000	29.4198	19.9663	.0000	1.5000	98.5604	93.3000	1170.7131	1.0144
27.9	.0000	1.5000	27.4198	20.8127	.0000	1.5000	98.2595	93.9347	1178.3873	1.0070
28.5	.0000	1.5000	25.4198	21.6591	.0000	1.5000	97.9585	94.5739	1183.3412	1.0023
29.3	.0000	1.5000	23.4198	22.5055	.0000	1.5000	97.6576	95.2173	1185.5750	1.0001
30.0	.0000	1.5000	22.3858	22.3858	.0000	1.5000	96.6109	96.6109	1185.0886	1.0000

Edited Data for Initial Conditions of
Central Velocity Ratio of 1.5 and Peripheral Radius Ratio of 3.0

X/r_{a_o}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	1.5000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	1.0222
11.7	.9000	1.0813	1.5007	1.0011	.8997	1.1063	100.0000	.0000	.0028	1.0209
18.1	.8000	1.1943	1.5000	1.0000	.7238	1.2525	100.0000	.0000	.0049	1.0199
22.6	.7000	1.3063	1.4991	.9987	.4992	1.3988	100.0000	.0000	.0069	1.0191
26.4	.6000	1.4193	1.4981	.9971	.0647	1.5450	100.0000	.0000	.0087	1.0183
29.8	.5000	1.5313	1.4969	.9953	.0000	1.6913	85.8086	.0000	.0104	1.0175
32.8	.4000	1.6443	1.4955	.9933	.0000	1.8375	73.7677	.0000	.0119	1.0168
35.4	.3000	1.7563	1.4940	.9909	.0000	1.9838	64.1954	.0000	.0132	1.0162
37.7	.2000	1.8491	1.4930	.9895	.0000	2.1038	57.1810	.0000	.0159	1.0150
39.6	.1000	1.9343	1.4924	.9886	.0000	2.2145	52.1133	.0000	.0177	1.0142
43.0	.0000	2.1047	1.4467	.9886	.0000	2.4360	43.9363	.0000	.0224	1.0121
46.1	.0000	2.2751	1.3816	.9882	.0000	2.6576	37.6743	.0000	.0282	1.0095
49.2	.0000	2.4455	1.3354	.9865	.0000	2.8791	32.5584	.0000	.0315	1.0080
52.0	.0000	2.6159	1.2966	.9850	.0000	3.0000	29.3300	.9996	.0343	1.0068
53.2	.0000	2.7011	1.2792	.9844	.0000	3.0000	28.3535	2.0774	.0356	1.0062
54.6	.0000	2.7863	1.2632	.9838	.0000	3.0000	27.4540	3.1552	.0368	1.0056
55.9	.0000	2.8715	1.2484	.9832	.0000	3.0000	26.6255	4.1525	.0380	1.0051
57.2	.0000	2.9567	1.2348	.9826	.0000	3.0000	25.8623	5.0281	.0391	1.0046
58.5	.0000	3.0000	1.1348	1.0219	.0000	3.0000	20.2438	10.7704	.0475	1.0008
60.0	.0000	3.0000	1.0349	1.0642	.0000	3.0000	14.6254	16.7642	.0497	.9999

Edited Data for Initial Conditions of
Central Velocity Ratio of 3.0 and Peripheral Radius Ratio of 3.0

X/r_{a_0}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	3.0000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	1.2645
12.2	.9000	1.1342	2.9971	.9912	.8726	1.1534	100.0000	.0000	.0224	1.2570
17.2	.8000	1.2686	2.9936	.9807	.7381	1.3070	100.0000	.0000	.0429	1.2501
21.2	.7000	1.4030	2.9897	.9688	.5651	1.4606	100.0000	.0000	.0612	1.2440
24.3	.6000	1.5330	2.9857	.9564	.3659	1.6091	100.0000	.0000	.0809	1.2374
27.0	.5000	1.6547	2.9822	.9451	.1222	1.7482	100.0000	.0000	.1023	1.2302
28.6	.4000	1.7727	2.9786	.9337	.0000	1.8831	93.7517	.0000	.1239	1.2230
31.4	.3000	1.8870	2.9750	.9222	.0000	2.0137	85.0029	.0000	.1453	1.2158
33.2	.2000	1.9978	2.9714	.9106	.0000	2.1403	77.7631	.0000	.1667	1.2087
35.0	.1000	2.1050	2.9678	.8989	.0000	2.2629	71.1400	.0000	.1879	1.2016
36.6	.0000	2.2122	2.9527	.8883	.0000	2.3854	66.0368	.0000	.2146	1.1926
39.8	.0000	2.4266	2.7282	.8583	.0000	2.6304	57.8961	.0000	.2667	1.1752
42.7	.0000	2.6410	2.5402	.8274	.0000	2.8754	51.4610	.0000	.3185	1.1579
45.4	.0000	2.8554	2.3800	.7952	.0000	3.0000	47.3503	2.0881	.3707	1.1404
48.1	.0000	3.0000	2.2087	.8046	.0000	3.0000	44.3797	6.9525	.4596	1.1106
50.8	.0000	3.0000	2.0087	.8893	.0000	3.0000	40.9114	11.3241	.5800	1.0703
53.6	.0000	3.0000	1.8087	.9739	.0000	3.0000	37.4430	15.6057	.6735	1.0390
56.1	.0000	3.0000	1.6087	1.0585	.0000	3.0000	33.9747	19.6394	.7398	1.0168
58.2	.0000	3.0000	1.4087	1.1432	.0000	3.0000	30.5064	23.4311	.7786	1.0039
60.0	.0000	3.0000	1.2087	1.2273	.0000	3.0000	27.0381	27.1824	.7905	.9999

Edited Data for Initial Conditions of
Central Velocity Ratio of 10.0 and Peripheral Radius Ratio of 3.0

X/r_{a_o}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	10.0000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	3.0000
10.8	.9000	1.1240	9.9980	.9797	.9053	1.1612	100.0000	.0000	.5210	2.9349
16.1	.8000	1.2760	9.9915	.9115	.7596	1.3588	100.0000	.0000	.6616	2.9173
20.1	.7000	1.4280	9.9847	.8329	.6018	1.5564	100.0000	.0000	.7584	2.9052
23.5	.6000	1.5800	9.9775	.7425	.4297	1.7540	100.0000	.0000	.8199	2.8975
26.3	.5000	1.7320	9.9707	.6446	.2564	1.9516	100.0000	.0000	.8312	2.8961
28.9	.4000	1.8840	9.9641	.5327	.0672	2.1492	100.0000	.0000	.8215	2.8973
31.3	.3000	2.0360	9.9584	.4109	.0000	2.3468	94.5858	.0000	.7777	2.9028
32.0	.2000	2.1880	9.9537	.2755	.0000	2.5444	87.9086	.0000	.7185	2.9102
34.8	.1000	2.3400	9.9507	.1254	.0000	2.7420	82.0368	.0000	.6523	2.9185
39.0	.0000	2.9480	8.6111	-.6766	.0000	3.0000	75.4231	16.7436	1.5881	2.8015
40.0	.0000	3.0000	8.0111	-.5440	.0000	3.0000	73.6200	22.7348	3.7139	2.5358
40.8	.0000	3.0000	7.4111	-.2901	.0000	3.0000	71.8169	27.0294	6.0443	2.2445
41.5	.0000	3.0000	6.8111	-.0362	.0000	3.0000	70.0138	31.0321	8.1299	1.9838
43.0	.0000	3.0000	6.0111	.3024	.0000	3.0000	67.6097	36.0124	10.5297	1.6838
45.5	.0000	3.0000	5.0111	.7256	.0000	3.0000	64.6045	41.4602	12.9174	1.3853
48.5	.0000	3.0000	4.0111	1.1488	.0000	3.0000	61.5993	46.0044	14.6252	1.1719
51.2	.0000	3.0000	3.0111	1.5720	.0000	3.0000	58.5942	50.2466	15.6529	1.0434
55.0	.0000	3.0000	2.5111	1.7836	.0000	3.0000	57.0916	52.5122	15.9117	1.0110
60.0	.0000	3.0000	1.9111	2.0375	.0000	3.0000	55.2885	55.0647	15.9980	1.0003

[illegible]

Edited Data for Initial Conditions of
Central Velocity Ratio of 50.0 and Peripheral Radius Ratio of 3.0

X/r_a	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	50.0000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	6.7099
9.9	.9000	1.1330	49.9966	.8147	.9025	1.1729	100.0000	.0000	1.3962	6.6931
14.8	.8000	1.2930	49.9914	.3752	.7643	1.3809	100.0000	.0000	2.7308	6.6770
18.3	.7000	1.4530	49.9898	-.1313	.6228	1.5889	100.0000	.0000	6.4985	6.6316
21.4	.6000	1.6130	49.9852	-.6946	.4813	1.7969	100.0000	.0000	6.7445	6.6287
24.0	.5000	1.7730	49.9724	-1.3275	.3429	2.0049	100.0000	.0000	9.6760	6.5934
26.4	.4000	1.9330	49.9489	-2.0274	.2154	2.2129	100.0000	.0000	11.9769	6.5657
28.5	.3000	2.0930	49.9113	-2.8036	.1102	2.4209	100.0000	.0000	12.5951	6.5582
30.2	.2000	2.2370	49.8625	-3.5685	.0532	2.6081	100.0000	.0000	13.0399	6.5529
32.0	.1000	2.3730	49.8101	-4.2372	.0198	2.7849	100.0000	.0000	25.0724	6.4080
34.7	.0000	3.0000	39.9713	-7.7442	.0000	3.0000	97.0369	47.0827	92.0672	5.6015
36.3	.0000	3.0000	35.0713	-5.6706	.0000	3.0000	95.4541	55.7897	195.6232	4.3547
38.5	.0000	3.0000	30.0713	-3.5546	.0000	3.0000	93.8390	63.5951	284.4612	3.2852
41.0	.0000	3.0000	25.9713	-1.8194	.0000	3.0000	92.6146	68.0910	344.6214	2.5609
45.0	.0000	3.0000	19.9713	.7198	.0000	3.0000	90.5764	73.6393	412.0559	1.7490
47.9	.0000	3.0000	16.9713	1.9893	.0000	3.0000	89.6073	76.1109	436.5939	1.4536
50.4	.0000	3.0000	13.9713	3.2589	.0000	3.0000	88.6383	79.6422	455.0102	1.2319
53.5	.0000	3.0000	10.9713	4.5285	.0000	3.0000	87.6692	80.8150	467.3061	1.0839
56.0	.0000	3.0000	8.9713	5.3749	.0000	3.0000	87.0231	82.3272	472.1031	1.0261
60.0	.0000	3.0000	6.4713	6.4329	.0000	3.0000	86.2156	84.1897	474.2740	1.0000

Edited Data for Initial Conditions of
Central Velocity Ratio of 1.5 and Peripheral Radius Ratio of 10.0

X/r_{a_o}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	1.5000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	1.0025
11.6	.9000	1.0300	1.5002	1.0003	.9806	1.0390	100.0000	.0000	.0005	1.0022
20.0	.7000	1.0900	1.5006	1.0010	.9442	1.1170	100.0000	.00C0	.0014	1.0018
40.1	.3000	1.3617	1.5009	1.0013	.4222	1.4701	100.0000	.00CC	.0024	1.0013
50.0	.1000	1.5609	1.5007	1.0010	.CCCC	1.7291	84.7126	.00C0	.0025	1.0012
58.1	.0000	1.7601	1.4450	1.0009	.CCCC	1.9881	65.2692	.C0CC	.0029	1.0010
68.6	.CCCC	2.0589	1.3387	1.0007	.CCCC	2.3765	47.2055	.00C0	.0033	1.0008
80.5	.0000	2.4573	1.2462	1.0006	.C0CC	2.8944	32.7236	.00CC	.0037	1.0006
91.0	.CCCC	2.8557	1.1885	1.0004	.CCC0	3.4123	23.9360	.00C0	.0040	1.0005
100.0	.0000	3.2541	1.1498	1.0003	.00CC	3.9303	18.2645	.C0CC	.0041	1.0004
110.8	.CCCC	3.7521	1.1128	1.0003	.CCCC	4.5777	13.6111	.CCCC	.0043	1.0003
120.2	.CC00	4.2501	1.0906	1.0001	.C0CC	5.2251	10.5225	.C0CC	.0045	1.0002
130.8	.C0CC	4.8477	1.0697	1.0001	.CCCC	6.0019	8.0189	.C0C0	.0046	1.0002
140.9	.C000	5.4453	1.0552	1.0001	.C0C0	6.7788	6.3177	.C0CC	.0047	1.0001
149.8	.C0CC	6.0429	1.0461	1.0000	.CCCC	7.5557	5.0981	.C0CC	.0047	1.0001
160.0	.C000	6.7401	1.0371	1.0000	.C0C0	8.4621	4.0688	.C0CC	.0048	1.0001
170.8	.C0CC	7.5369	1.0296	1.0000	.CCCC	9.4979	3.2448	.00C0	.0048	1.0001
179.5	.CCCC	8.2341	1.0248	1.0000	.CCCC	10.0000	2.8166	.1115	.0049	1.0000
190.2	.CCCC	9.1305	1.0202	1.0000	.CCCC	10.0000	2.5074	.4347	.0049	1.0000
200.0	.CCCC	10.0000	1.0171	1.0000	.CCCC	10.0000	2.3020	.6334	.0049	1.0000

Edited Data for Initial Conditions of
Central Velocity Ratio of 3.0 and Peripheral Radius Ratio of 10.0

X/r_{a_o}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MCMENTUM
0.0	1.00C0	1.0000	3.0000	1.0000	1.0000	1.0000	1C0.CC00	.0C00	.0000	1.C381
19.6	.9000	1.0880	3.0001	1.0002	.928C	1.1144	100.0000	.0000	.0045	1.0359
31.0	.80C0	1.2211	2.9998	.9994	.7641	1.2874	1C0.CC00	.0C00	.0061	1.0351
39.3	.70C0	1.3542	2.9995	.9985	.5669	1.4605	1C0.C000	.0000	.0075	1.0345
52.0	.50C0	1.6204	2.9988	.9963	.CCCC	1.8065	98.3064	.CCCC	.0093	1.0336
62.1	.30C0	1.8866	2.9979	.9937	.CC00	2.1526	74.5285	.CCCC	.0098	1.0334
69.7	.10C0	2.1120	2.9972	.9916	.CCCC	2.4456	60.4734	.CCCC	.0118	1.0324
81.3	.00CC	2.5164	2.5923	.9898	.C000	2.9713	46.0103	.CCCC	.025C	1.0260
91.3	.C0C0	2.9208	2.2984	.9867	.CCCC	3.4970	35.219C	.CCCC	.0309	1.0232
100.7	.COCC	3.3252	2.0781	.9840	.C000	4.0228	28.0867	.CCCC	.0362	1.0207
110.0	.00C0	3.8307	1.8721	.9811	.CCCC	4.6799	21.8323	.CCCC	.0421	1.0178
120.3	.COCC	4.3362	1.7203	.9785	.C000	5.3371	17.4372	.CCC0	.0470	1.0155
131.3	.COC0	4.9428	1.5826	.9759	.CCCC	6.1256	13.7198	.CCCC	.0521	1.0130
139.7	.COCC	5.4483	1.4939	.9741	.C000	6.7828	11.4663	.CCCC	.0558	1.0113
150.0	.C0C0	6.1560	1.3993	.9718	.CCCC	7.7028	9.1290	.CCC0	.06CC	1.0092
160.2	.COCC	6.8637	1.3268	.9700	.C0CC	8.6228	7.4293	.CCCC	.0635	1.0075
170.0	.COC0	7.5714	1.2705	.9685	.CCCC	9.5428	6.1664	.0000	.0665	1.0061
181.2	.COCC	8.4813	1.2151	.9669	.C000	10.0000	5.2914	.3924	.0695	1.0047
190.1	.COCC	9.2901	1.1772	.9657	.CCCC	10.000C	4.8347	.9263	.0718	1.0036
200.0	.CCCC	10.0000	1.0502	1.0072	.CC00	10.0000	3.3052	2.4870	.0789	1.0002

Edited Data for Initial Conditions of
Central Velocity Ratio of 10.0 and Peripheral Radius ratio of 10.0

X/r_{a_0}	RA	RB	VP	VS	RAT	RBT	TP	TS	DP	MOMENTUM
0.0	1.0000	1.0000	10.0000	1.0000	1.0000	1.0000	100.0000	.0000	.0000	1.6749
22.6	.9000	1.1519	9.9995	.9950	.8698	1.1823	100.0000	.0000	.0148	1.6687
32.0	.8000	1.3033	9.9989	.9892	.7347	1.3646	100.0000	.0000	.0233	1.6651
39.0	.7000	1.4530	9.9983	.9831	.5798	1.5436	100.0000	.0000	.0287	1.6628
49.4	.5000	1.7275	9.9972	.9716	.2687	1.8731	100.0000	.0000	.0513	1.6533
61.2	.2000	2.1055	9.9955	.9543	.0000	2.3266	88.3180	.0000	.0848	1.6393
70.0	.0000	2.4574	9.4966	.9363	.0000	2.7489	71.6688	.0000	.1257	1.6221
80.9	.0000	2.9265	8.0202	.9089	.0000	3.3119	57.5169	.0000	.1763	1.6007
90.0	.0000	3.3953	6.9540	.8818	.0000	3.8750	47.6419	.0000	.2249	1.5803
102.2	.0000	4.0995	5.8142	.8415	.0000	4.7195	37.4954	.0000	.2948	1.5509
109.9	.0000	4.5683	5.2518	.8146	.0000	5.2826	32.6706	.0000	.3388	1.5323
120.2	.0000	5.2725	4.5968	.7740	.0000	6.1271	27.2208	.0000	.4030	1.5053
130.0	.0000	5.9764	4.0970	.7327	.0000	6.9717	23.2470	.0000	.4656	1.4790
140.0	.0000	6.7975	3.6472	.6827	.0000	7.9570	19.8283	.0000	.5359	1.4494
150.0	.0000	7.6185	3.2958	.6299	.0000	8.9423	17.2912	.0000	.6052	1.4202
160.0	.0000	8.5570	2.9803	.5639	.0000	10.0000	15.2695	.0331	.6835	1.3873
170.0	.0000	9.4954	2.7324	.4884	.0000	10.0000	14.4701	1.8733	.7626	1.3540
180.0	.0000	10.0000	2.2293	.6078	.0000	10.0000	12.8481	4.5093	1.1624	1.1858
190.2	.0000	10.0000	1.6293	.8618	.0000	10.0000	10.9134	7.0506	1.5049	1.0416
200.0	.0000	10.0000	1.1293	1.0733	.0000	10.0000	9.3012	8.9306	1.6033	1.0002